Week 25

A-Level Physics

Homework bundle for this week

本周作业合集

exercises.lol

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12. Motion in a circle

Starter Quiz · 课前小测

A-Level Physics   Name: ____________________   Score: ________

1. One radian is the angle for which the arc length equals the:

   ☐ radius

   ☐ diameter

   ☐ circumference

   ☐ area

2. An object moving round a circle at constant speed is still accelerating because:

   ☐ its velocity keeps changing direction

   ☐ its speed keeps increasing

   ☐ it is slowing down

   ☐ no resultant force acts

3. How many radians are there in a complete circle?

   ______________ rad

4. An object moves at 4.0 $\frac{\text{m}}{\text{s}}$ round a circle of radius 2.0 m. Find its centripetal acceleration.

   ______________ m/s^2

5. An object goes once round every 2.0 s. What is its angular speed?

   ______________ rad/s

6. The centripetal force points toward the centre of the circle.

   ☐ True   ☐ False

7. Angular speed equals 2π divided by the ______.

   ____

8. A 2.0 kg ball moves at 4.0 $\frac{\text{m}}{\text{s}}$ round a circle of radius 2.0 m. What centripetal force is needed?

   ______________ N

9. A point at radius 0.50 m turns at $\omega = 4.0\ \frac{\text{rad}}{\text{s}}$. What is its linear speed?

   ______________ m/s

10. The centripetal force is:

    ☐ the net result of real forces (tension, gra friction…)

- ☐ a brand-new kind of force
- ☐ always friction
- ☐ always zero

A-Level Physics

1. **radius**

   $\theta = \frac{s}{r}$, so when the arc s equals the radius r, the angle is exactly one radian.

2. **its velocity keeps changing direction**

   Velocity is a vector. Even at constant speed, the changing direction means a changing velocity — an acceleration toward the centre.

3. **6.28 rad**

   A full circle has arc $s = 2\pi r$, so $\theta = \frac{2\pi r}{r} = 2\pi \approx 6.28$ rad.

4. **8 m/s^2**

   $a = \frac{v^2}{r} = \frac{4.0^2}{2.0} = 8.0\ \frac{\mathrm{m}}{\mathrm{s}^2}$.

5. **3.14 rad/s**

   $\omega = \frac{2\pi}{T} = \frac{2\pi}{2.0} = \pi \approx 3.14\ \frac{\mathrm{rad}}{\mathrm{s}}$.

6. **True**

   Yes — always toward the centre, perpendicular to the velocity.

7. **period**

   $\omega = \frac{2\pi}{T}$ — the whole turn (2π) divided by the time for one turn.

8. **16 N**

   $F = \frac{mv^2}{r} = \frac{2.0 \times 4.0^2}{2.0} = 16$ N.

9. **2 m/s**

   $v = r\omega = 0.50 \times 4.0 = 2.0\ \frac{\mathrm{m}}{\mathrm{s}}$.

10. **the net result of real forces (tension, gravity, friction...)**

    It is whatever real force(s) happen to point to the centre — not a separate force of its own.

A-LEVEL PHYSICS • EXERCISE SHEET

12.1 Kinematics of uniform circular motion

Name: ____________ Class: ________ Date: __________ **Total: 14 marks**

KEY WORDS angular speed 角速度 • radian 弧度 • period 周期 • arc 弧 • revolution 圈

Objective

Build the skills to answer exam questions on **circular motion kinematics** — radians, angular speed, $v = r\omega$, and **connected** wheels.

You must be able to:

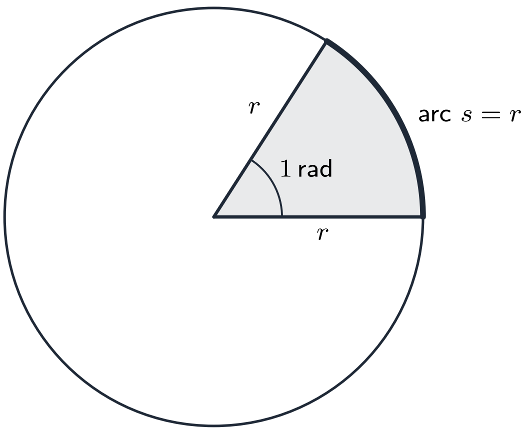
- define the **radian** and convert between radians and degrees
- use **angular speed** $\omega = \theta/t = 2\pi/T = 2\pi f$ and $v = r\omega$
- handle **connected wheels** (same axle → same ω; linked by a belt/chain → same rim speed)

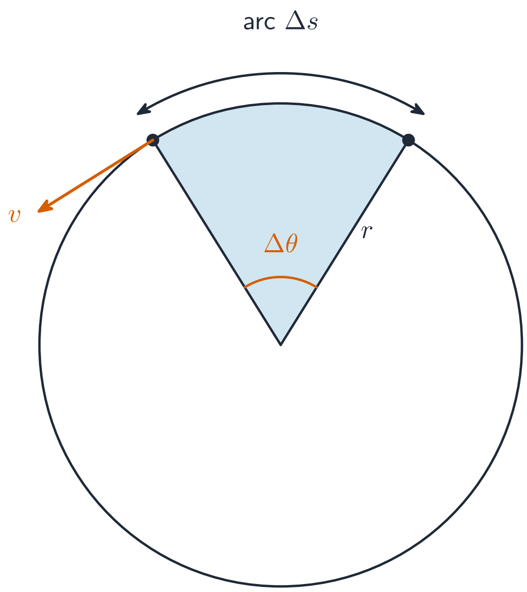
1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Radians

One **radian** is the angle whose arc length equals the radius: $\theta = s/r$:





Angular speed in uniform circular motion

A full circle = 2π rad; 1 rad = $180°/\pi \approx 57.3°$. **Use radian mode** on your calculator.

Angular speed

$$\omega = \frac{\theta}{t} = \frac{2\pi}{T} = 2\pi f \quad (\text{rad s}^{-1}), \qquad v = r\omega.$$

Worked example. A point on the equator (radius 6.4×10^6 m, period 24 h): $\omega = \dfrac{2\pi}{24 \times 3600} = 7.3 \times 10^{-5}$ rad s^{-1}; $v = r\omega = 465$ m s^{-1}.

Connected wheels

Exam questions often link wheels, cogs or pulleys. Two rules:

- **Fixed on the same axle** → they turn with the **same** ω (but different rim speeds if the radii differ).
- **Linked by a belt or chain** (no slipping) → the **rim speeds are equal**: $r_1\omega_1 = r_2\omega_2$.

Worked example. A belt links pulley A (radius 0.10 m, $\omega_A = 30$ rad s^{-1}) to pulley B (radius 0.050 m). Rim speed $v = r_A\omega_A = 0.10 \times 30 = 3.0$ m s^{-1}; so $\omega_B = \dfrac{v}{r_B} = \dfrac{3.0}{0.050} = 60$ rad s^{-1} (the smaller pulley spins faster).

2 Practice

Now apply the methods above.

2.1 Define the **radian**. [1]

2.2 Convert 90° to **radians**. [1]

2.3 A wheel turns at 2.0 revolutions per second. Find its **angular speed**. [2]

2.4 Two pulleys are linked by a belt. State what is **equal** for the two pulleys. [1]

3 Exam-style questions

3.1 A full circle, in radians, is: [1]

- **A** π
- **B** 2π
- **C** 360
- **D** 1

3.2 A car travels around a circular track of radius 50 m at 20 m s^{-1}. Find its **angular speed**. [2]

3.3 In a workshop, a motor turns a pulley of radius 25 mm at 1500 revolutions per minute. A tight belt runs from that pulley to a larger pulley of radius 75 mm on the shaft of a fan.

(a) Determine the angular speed of the **motor pulley**, in rad s^{-1}. [2]

(b) The belt does not slip. Determine the angular speed of the **fan**, and state what stays the same for both pulleys. [3]

3.4 Convert 1.5 rad to **degrees**. [1]

Solutions

2.1 The angle subtended at the centre of a circle by an arc whose **length equals the radius**.

2.2 $90° = \frac{\pi}{2}$ rad ≈ 1.57 rad.

2.3 $\omega = 2\pi f = 2\pi \times 2.0 = 4\pi = 12.6$ rad s^{-1}.

2.4 Their **rim speeds** ($v = r\omega$) are equal (the belt does not slip).

3.1 **B** — 2π.

3.2 $\omega = v/r = 20/50 = 0.40$ rad s^{-1}.

3.3 (a) 1500 revolutions per minute = 25 revolutions per second, and one revolution is 2π rad, so $\omega = 25 \times 2\pi = 157$ rad s^{-1}.

3.3 (b) The belt gives both pulleys the same **rim speed** $v = r\omega$: $v = 0.025 \times 157 = 3.93$ m s^{-1}, so the fan turns at $\omega = \frac{3.93}{0.075} = 52$ rad s^{-1} — a third of the motor's, because its radius is three times larger.

3.4 $1.5 \times \frac{180}{\pi} = 86°$.

1 **(a)** In terms of velocity and acceleration, describe uniform circular motion of an object.

....

....

.... [2]

(b) Fig. 1.1 shows the view from above of a polystyrene ball undergoing horizontal circular motion of radius R.

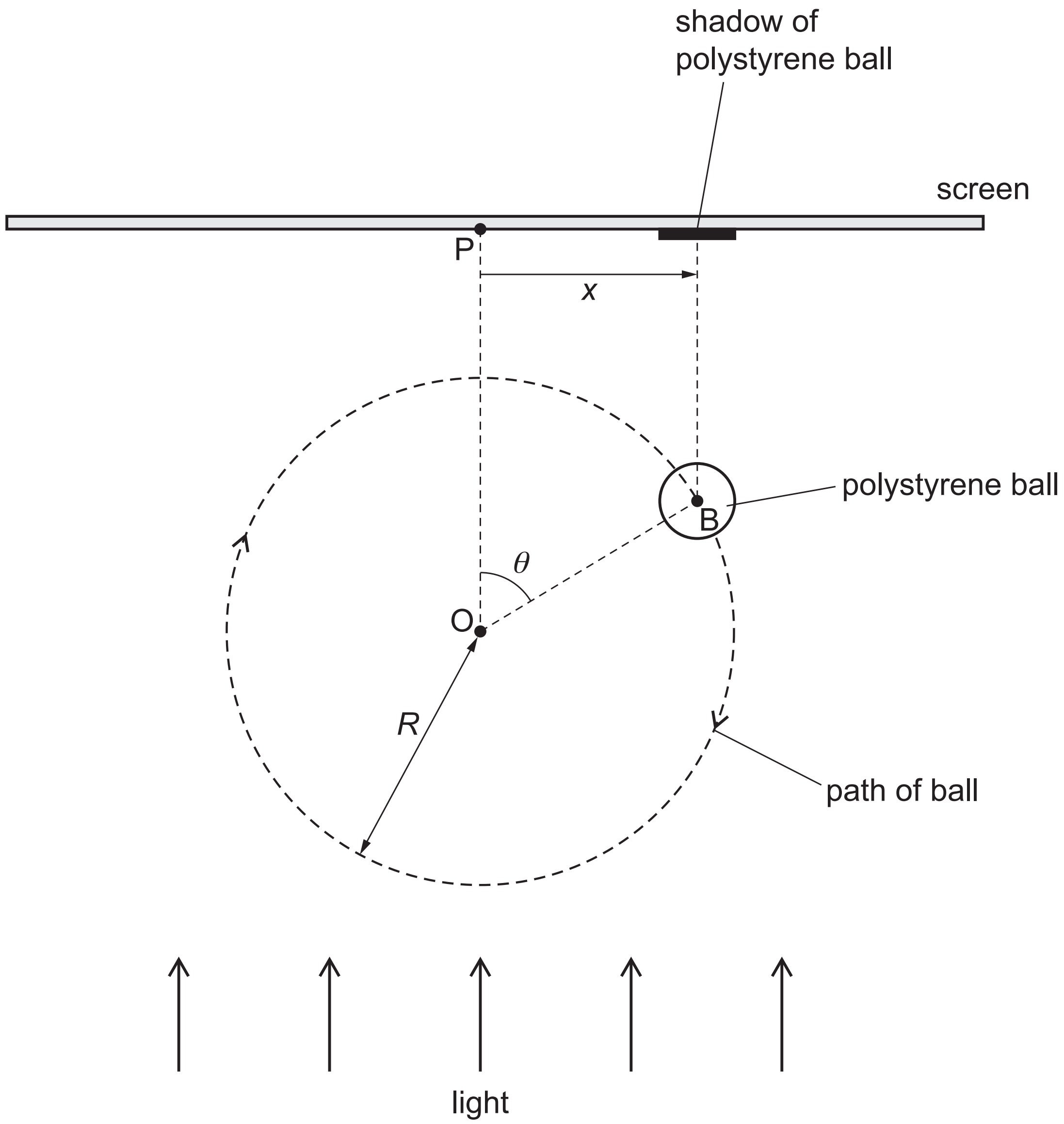


Fig. 1.1

The ball is illuminated by parallel light so that a shadow of the ball forms on a screen placed on the opposite side of the ball from the light source.

The line joining points O and P is perpendicular to the screen.

The angular speed of the circular motion is ω.

(i) State an expression, in terms of R and ω, for the speed v of the ball.

v = ... [1]

(ii) Determine an expression, in terms of v and ω, for the centripetal acceleration of the ball.

centripetal acceleration = ... [2]

(c) The ball in **(b)** is in the position shown in Fig. 1.1, such that line OB is at an angle θ to the line OP.

(i) Determine an expression, in terms of R and θ, for the displacement x of the shadow from P.

x = ... [1]

(ii) The value of θ is zero at time $t = 0$.

State an expression for θ in terms of ω and t.

θ = ... [1]

(iii) Use your answers in **(c)(i)** and **(c)(ii)** to show that x is given by

$$x = R \sin \omega t.$$

[1]

(iv) Explain, with reference to the equation in **(c)(iii)**, why the motion of the shadow of the ball on the screen may be modelled as simple harmonic.

....

[1]

(d) The circular motion of the ball in Fig. 1.1 has a diameter of 0.46 m and an angular speed of 1.9 rad s^{-1}.

For the simple harmonic motion of the shadow of the ball in Fig. 1.1, calculate:

(i) the amplitude

amplitude = ... m [1]

(ii) the period

period = ... s [2]

(iii) the maximum acceleration.

maximum acceleration = ... m s^{-2} [2]

(e) On Fig. 1.1, draw, and label with the letter A, the position of the shadow on the screen when the shadow has its maximum positive acceleration. [1]

[Total: 15]

1 The Earth may be considered as a uniform sphere of radius 6.37×10^{6} m.

Cambridge is at a point on the Earth's surface that has a latitude of 52.2° north of the Equator, as shown in Fig. 1.1.

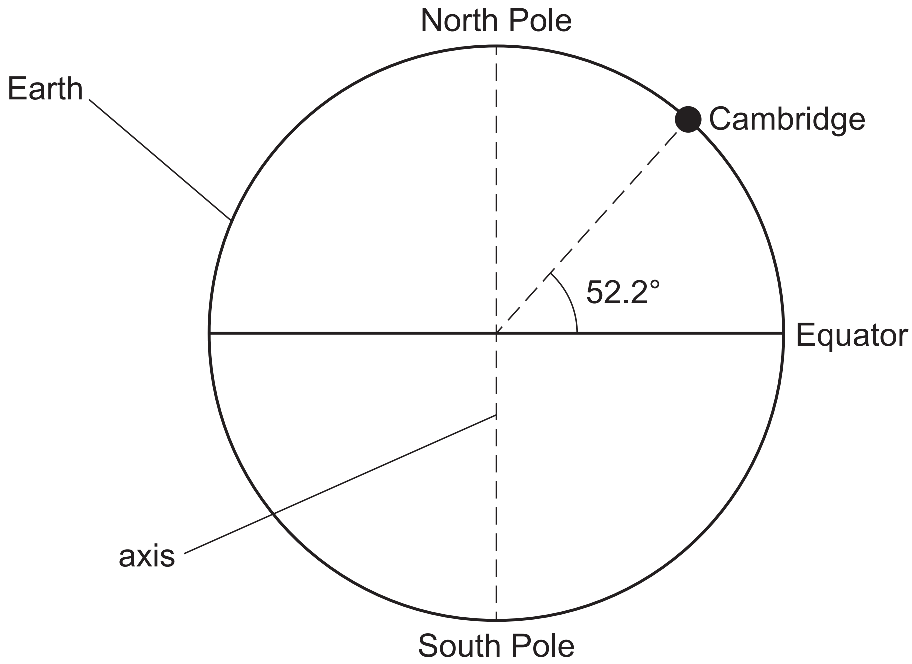


Fig. 1.1

As the Earth spins on its axis, Cambridge moves in a circle that is parallel to the Equator but with a smaller radius.

(a) (i) Show that the radius of the circle around which Cambridge moves is 3.90×10^{6} m.

[1]

(ii) Calculate the speed at which Cambridge moves around the circle.

speed = ... $m\,s^{-1}$ [3]

(b) A student of mass 58.6 kg stands on horizontal ground in Cambridge.

(i) Determine the magnitude of the resultant force that acts to cause the circular motion of the student.

resultant force = ... N [2]

(ii) On Fig. 1.2, draw an arrow to show the direction of the resultant force that acts on the student.

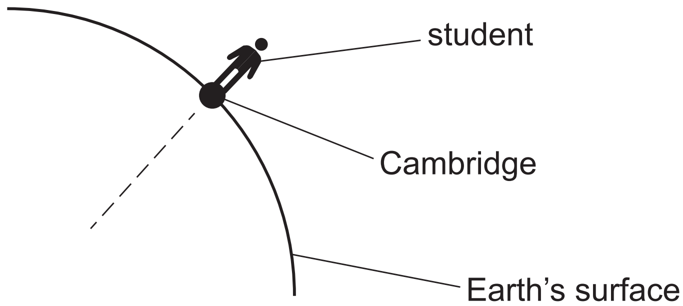


Fig. 1.2 (not to scale)

[1]

(iii) On Fig. 1.3, draw labelled arrows from the student to show the directions of the forces that act on the student to cause the resultant force in **(b)(ii)**.

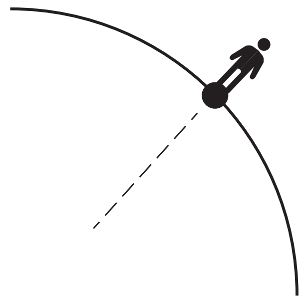


Fig. 1.3 (not to scale)

[2]

[Total: 9]

1 (a) Define the radian.

....

.... [1]

(b) The rear wheel and the pedals of a bicycle are connected by a chain that passes around two cogs (toothed wheels), as shown in Fig. 1.1.

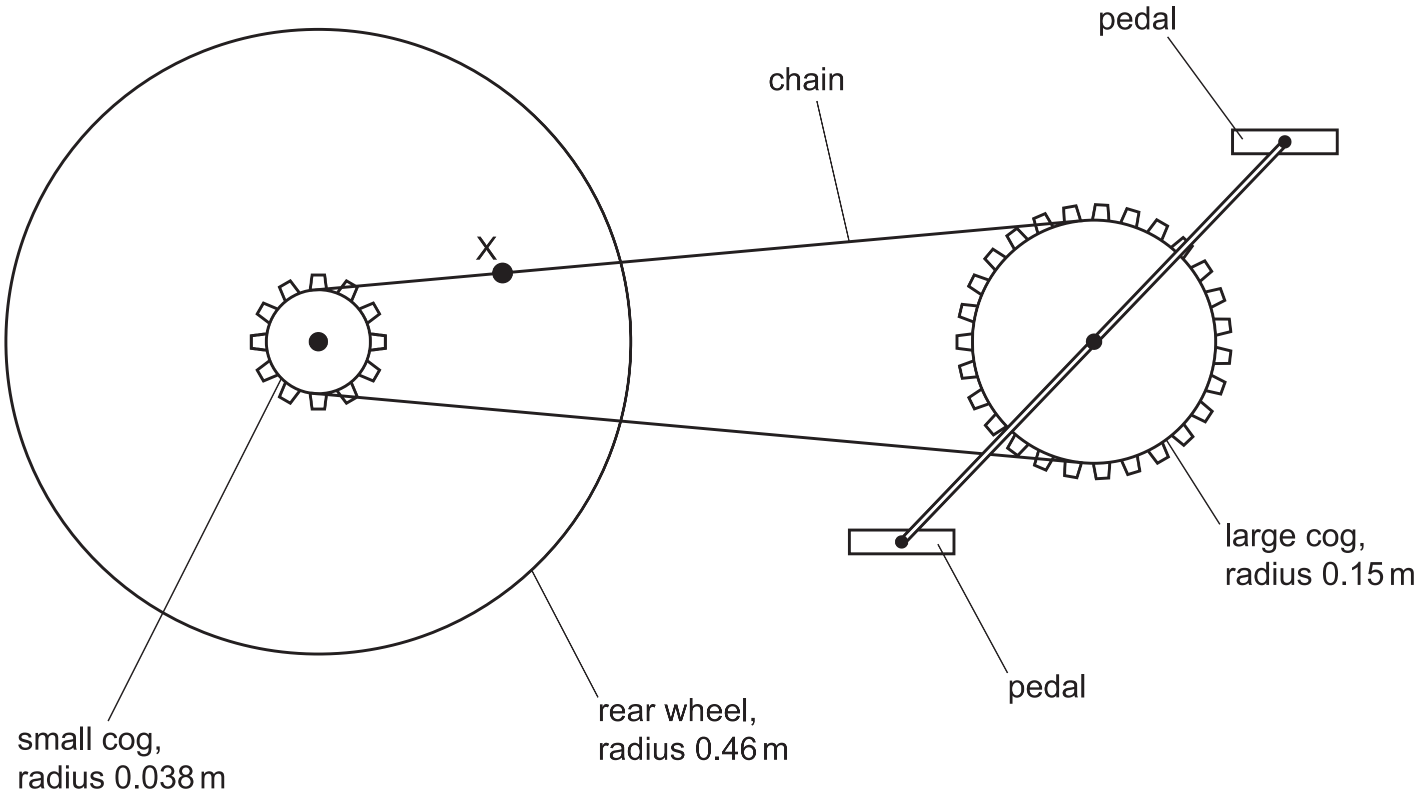


Fig. 1.1 (not to scale)

The small cog has a radius of 0.038 m and is fixed to the rear wheel so that it rotates with it. The large cog has a radius of 0.15 m and is fixed to the pedals so that it rotates with them. The rear wheel has a radius of 0.46 m.

The bicycle is being pedalled so that it moves in a straight line at a constant speed of 17 m s^{-1}.

(i) Calculate the angular speed of the rear wheel.

angular speed = ... rad s^{-1} [2]

(ii) Calculate the period of rotation of the small cog.

period = ... s [2]

(iii) Show that the distance moved by point X on the chain during one full rotation of the small cog is 0.24 m.

[1]

(iv) Use the information in **(b)(iii)** to determine the angle through which the large cog rotates during one full rotation of the small cog.

angle = ... rad [2]

(c) The chain of the bicycle in **(b)** is moved onto a smaller cog fixed to the rear wheel. The speed of the bicycle does not change.

Explain, without calculation, the effect of this change on the angular speed of the pedals.

....

....

.... [2]

[Total: 10]

1 A steel ball is placed on the inside surface of a hollow circular cone. The ball moves in a horizontal circle at constant speed, as shown in Fig. 1.1.

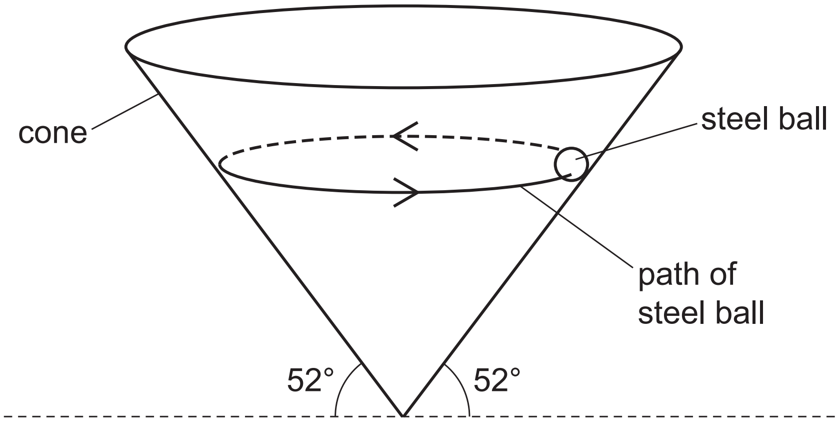


Fig. 1.1

The angle of the side of the cone to the horizontal is 52°. There is no friction between the ball and the cone.

(a) Fig. 1.2 shows a cross-section through the cone and the steel ball.

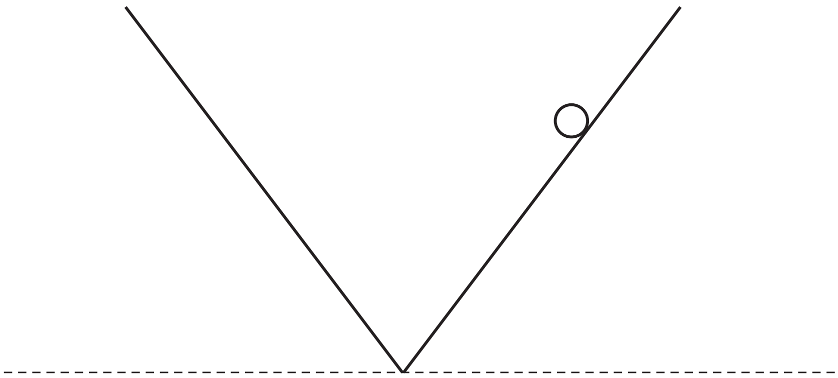


Fig. 1.2

On Fig. 1.2, draw labelled arrows to show the **two** forces acting on the ball. [1]

(b) Describe how the forces acting on the ball cause its acceleration to be centripetal.

....

....

....

.... [2]

(c) The ball moves in a circle of radius 0.15 m.

Show that the speed of the ball is $1.4\,m\,s^{-1}$.

[3]

(d) Calculate the angular speed ω of the ball.

ω = ... $rad\,s^{-1}$ [2]

(e) The speed of the ball is increased.

Explain why the radius of the circular path of the ball increases.

....

....

.... [1]

[Total: 9]

12.2 Centripetal acceleration

Name: ____________ Class: ________ Date: __________ **Total: 18 marks**

KEY WORDS centripetal acceleration 向心加速度 • centripetal force 向心力 • acceleration 加速度 • force 力 • friction 摩擦力

Objective

Build the skills to answer exam questions on **centripetal acceleration and force**.

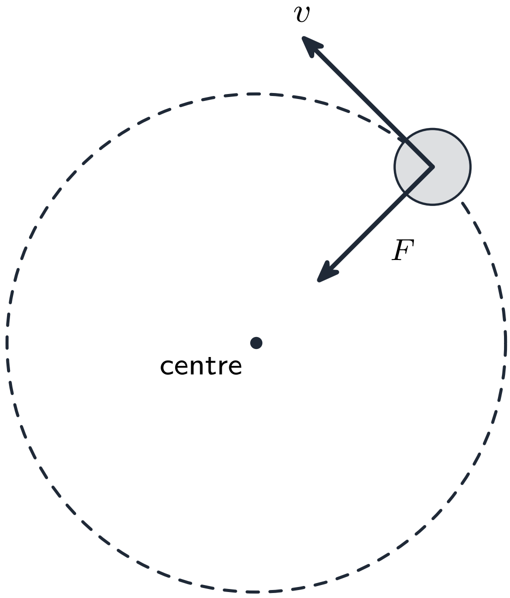
You must be able to:

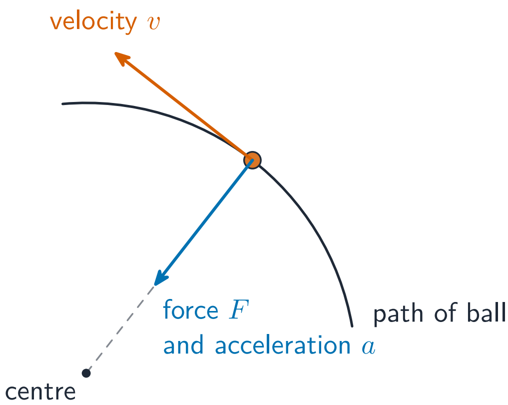
- describe uniform circular motion in terms of **velocity** and **acceleration**
- use $a = v^2/r = r\omega^2$ and $F = mv^2/r = mr\omega^2$
- identify the **real force** that provides the centripetal force (tension, friction, gravity, electric)

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

> ### ■ Centripetal acceleration
> In a circle at constant speed, the **speed** is constant but the **velocity** keeps changing direction — so there is an acceleration, directed **towards the centre**, perpendicular to v:





Centripetal acceleration points to the centre

$$a = \frac{v^2}{r} = r\omega^2.$$

■ Centripetal force

$$F = ma = \frac{mv^2}{r} = mr\omega^2.$$

This is **not a new force** — it is provided by a **real** force acting towards the centre: **tension** (string), **friction** (a car cornering), **gravity** (a satellite or planet), or the **electric force** (an electron orbiting a nucleus).

Worked example. A 0.20 kg ball whirled on a string in a 0.50 m horizontal circle at 3.0 m s^{-1}: the string tension provides $F = \dfrac{0.20 \times 3.0^2}{0.50} = 3.6$ N.

Worked example (orbit). If an electric force of 8.2×10^{-8} N keeps an electron (mass 9.1×10^{-31} kg) in a circle of radius 5.3×10^{-11} m, that force is the centripetal force: $v = \sqrt{\dfrac{Fr}{m}} = \sqrt{\dfrac{(8.2 \times 10^{-8})(5.3 \times 10^{-11})}{9.1 \times 10^{-31}}} = 2.2 \times 10^{6}\ \mathrm{m\ s^{-1}}$.

2 Practice

Now apply the methods above.

2.1 A car drives round a roundabout at a steady 12 m s^{-1}. Explain why the car is **accelerating** even though its speed never changes, and state the direction of that acceleration. [3]

2.2 Write **two** forms of the centripetal acceleration equation. [2]

2.3 A 0.20 kg ball is whirled in a horizontal circle of radius 0.50 m at 3.0 m s^{-1}. Find the **centripetal force**. [2]

2.4 State what provides the centripetal force for a car turning a **flat** corner. [1]

3 Exam-style questions

3.1 For an object in uniform circular motion, the centripetal force is directed: [1]

- **A** along the velocity
- **B** towards the centre
- **C** away from the centre
- **D** opposite to the velocity

3.2 A car of mass 900 kg turns a corner of radius 30 m at 15 m s^{-1}. Calculate the **centripetal force** needed. [3]

3.3 An electron of mass 9.1×10^{-31} kg moves in a circle of radius 5.3×10^{-11} m. The electric force on it is 8.2×10^{-8} N and provides the centripetal force.

(a) State why the electron is **accelerating** even though its speed is constant. [1]

(b) Calculate the **speed** of the electron. [3]

3.4 A stone is whirled on a string in a horizontal circle. State what provides the centripetal force, and describe its motion **if the string breaks**. [2]

Solutions

2.1 Velocity is a **vector**, so it changes whenever the **direction** of travel changes, even at constant speed; a changing velocity means the car is accelerating. That acceleration is directed towards the **centre** of the roundabout, at right angles to the velocity.

2.2 $a = \dfrac{v^2}{r}$ and $a = r\omega^2$.

2.3 $F = \dfrac{mv^2}{r} = \dfrac{0.20 \times 3.0^2}{0.50} = 3.6$ N.

2.4 **Friction** between the tyres and the road.

3.1 **B** — towards the centre.

3.2 $F = \dfrac{mv^2}{r} = \dfrac{900 \times 15^2}{30} = 6750 \text{ N} \approx 6.8 \times 10^3$ N.

3.3 (a) Its **velocity is changing direction**, and a changing velocity is an acceleration (towards the centre).

3.3 (b) The electric force is the centripetal force: $F = \dfrac{mv^2}{r}$, so $v = \sqrt{\dfrac{Fr}{m}} = \sqrt{\dfrac{(8.2 \times 10^{-8})(5.3 \times 10^{-11})}{9.1 \times 10^{-31}}}$ $= 2.2 \times 10^6 \text{ m s}^{-1}$.

3.4 The **tension** in the string provides it; if the string breaks the stone flies off along a **straight line (the tangent)**, since there is no longer a centripetal force.

1 **(a)** In terms of velocity and acceleration, describe uniform circular motion of an object.

....

....

.... [2]

(b) Fig. 1.1 shows the view from above of a polystyrene ball undergoing horizontal circular motion of radius R.

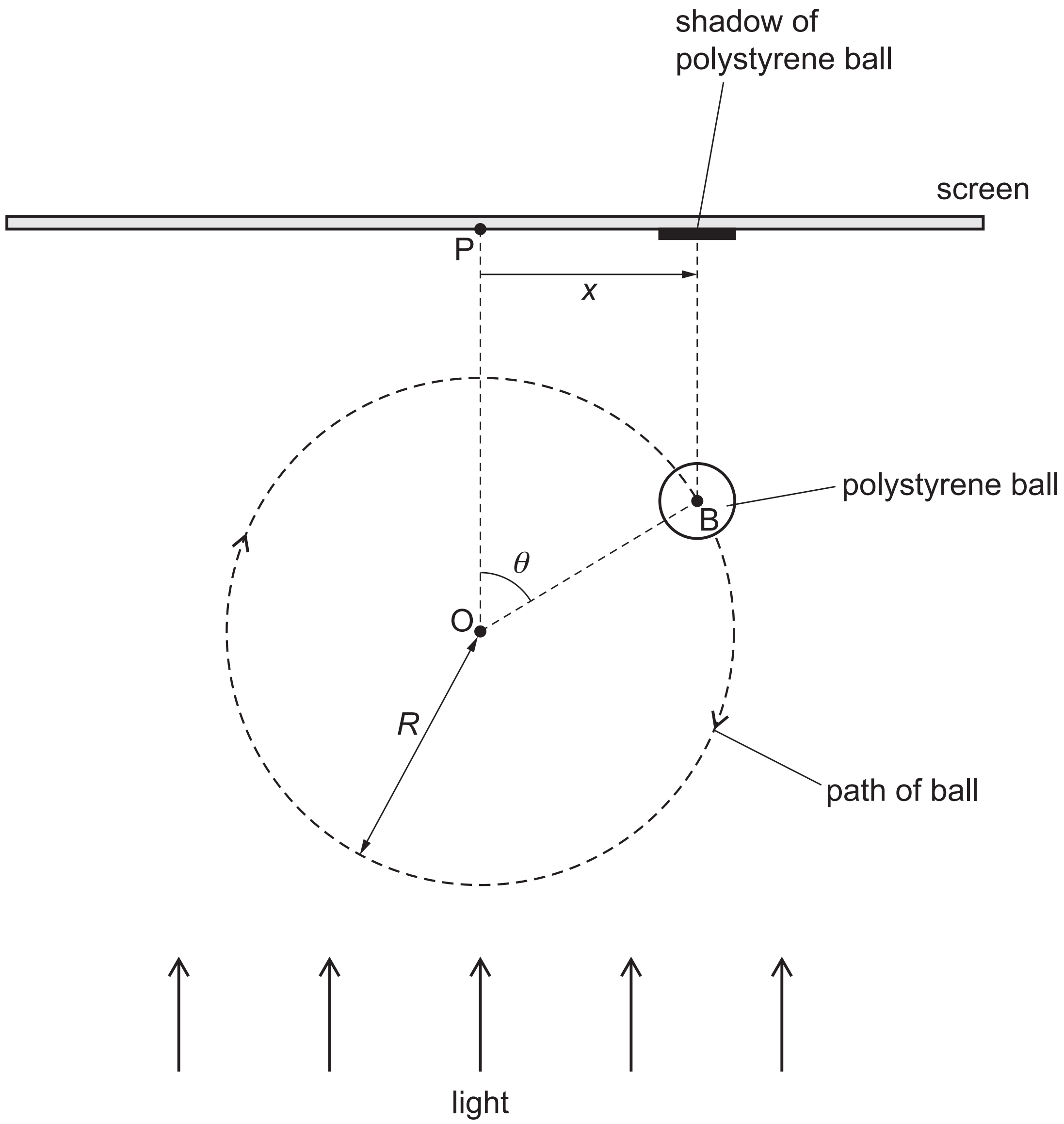


Fig. 1.1

The ball is illuminated by parallel light so that a shadow of the ball forms on a screen placed on the opposite side of the ball from the light source.

The line joining points O and P is perpendicular to the screen.

The angular speed of the circular motion is ω.

(i) State an expression, in terms of R and ω, for the speed v of the ball.

v = ... [1]

(ii) Determine an expression, in terms of v and ω, for the centripetal acceleration of the ball.

centripetal acceleration = ... [2]

(c) The ball in **(b)** is in the position shown in Fig. 1.1, such that line OB is at an angle θ to the line OP.

(i) Determine an expression, in terms of R and θ, for the displacement x of the shadow from P.

x = ... [1]

(ii) The value of θ is zero at time $t = 0$.

State an expression for θ in terms of ω and t.

θ = ... [1]

(iii) Use your answers in **(c)(i)** and **(c)(ii)** to show that x is given by

$$x = R \sin \omega t.$$

[1]

(iv) Explain, with reference to the equation in **(c)(iii)**, why the motion of the shadow of the ball on the screen may be modelled as simple harmonic.

....

[1]

(d) The circular motion of the ball in Fig. 1.1 has a diameter of 0.46 m and an angular speed of 1.9 rad s^{-1}.

For the simple harmonic motion of the shadow of the ball in Fig. 1.1, calculate:

(i) the amplitude

amplitude = ... m [1]

(ii) the period

period = ... s [2]

(iii) the maximum acceleration.

maximum acceleration = ... m s^{-2} [2]

(e) On Fig. 1.1, draw, and label with the letter A, the position of the shadow on the screen when the shadow has its maximum positive acceleration. [1]

[Total: 15]

1 The Earth may be considered as a uniform sphere of radius 6.37×10^6 m.

Cambridge is at a point on the Earth's surface that has a latitude of 52.2° north of the Equator, as shown in Fig. 1.1.

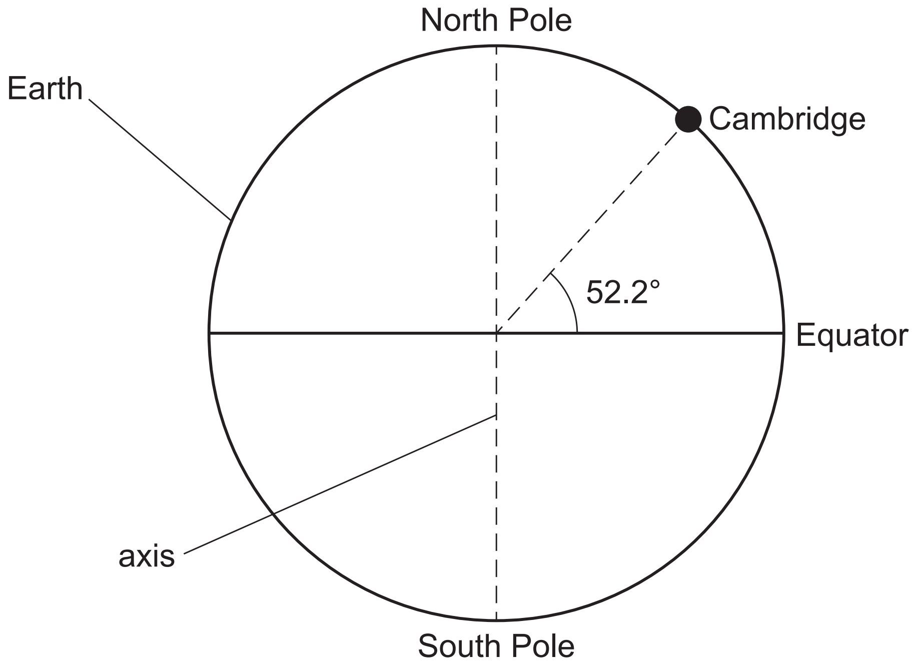


Fig. 1.1

As the Earth spins on its axis, Cambridge moves in a circle that is parallel to the Equator but with a smaller radius.

(a) (i) Show that the radius of the circle around which Cambridge moves is 3.90×10^6 m.

[1]

(ii) Calculate the speed at which Cambridge moves around the circle.

speed = ... $m s^{-1}$ [3]

(b) A student of mass 58.6 kg stands on horizontal ground in Cambridge.

(i) Determine the magnitude of the resultant force that acts to cause the circular motion of the student.

resultant force = ... N [2]

(ii) On Fig. 1.2, draw an arrow to show the direction of the resultant force that acts on the student.

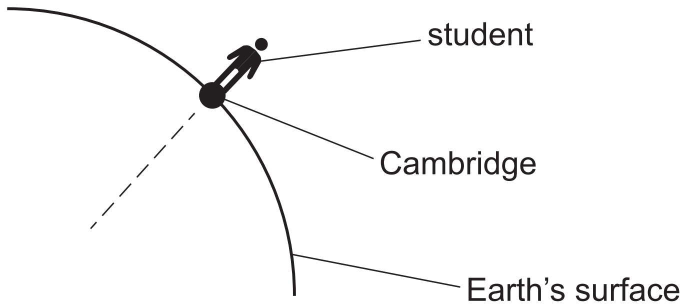


Fig. 1.2 (not to scale)

[1]

(iii) On Fig. 1.3, draw labelled arrows from the student to show the directions of the forces that act on the student to cause the resultant force in **(b)(ii)**.

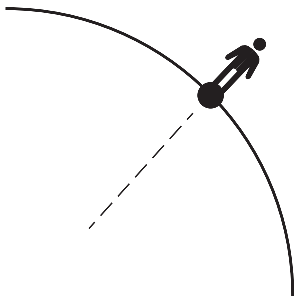


Fig. 1.3 (not to scale)

[2]

[Total: 9]

1 A steel ball is placed on the inside surface of a hollow circular cone. The ball moves in a horizontal circle at constant speed, as shown in Fig. 1.1.

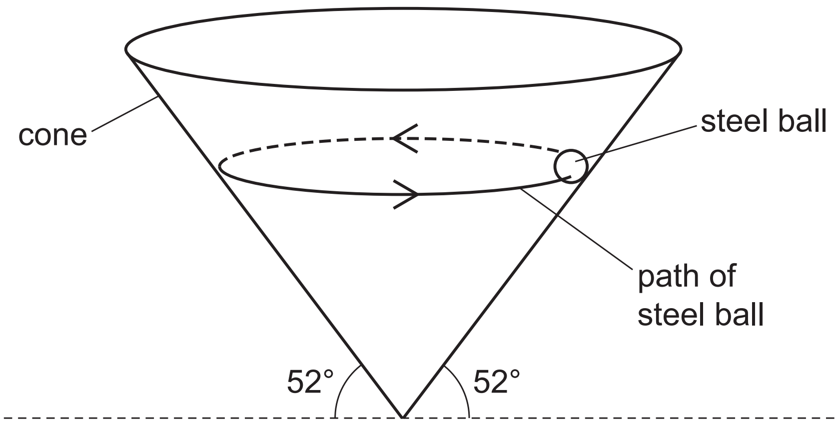


Fig. 1.1

The angle of the side of the cone to the horizontal is 52°. There is no friction between the ball and the cone.

(a) Fig. 1.2 shows a cross-section through the cone and the steel ball.

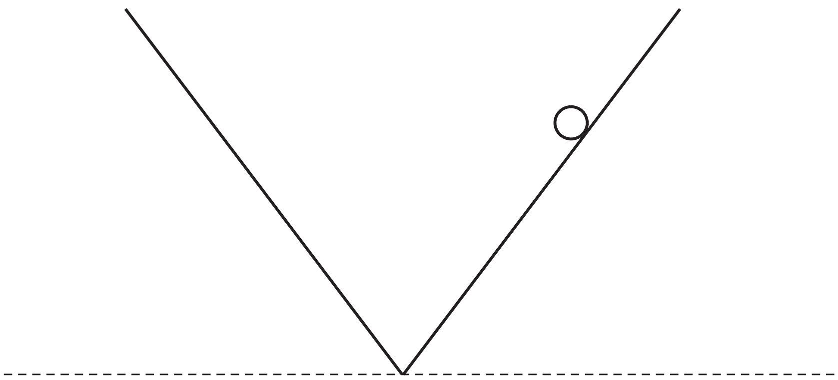


Fig. 1.2

On Fig. 1.2, draw labelled arrows to show the **two** forces acting on the ball. [1]

(b) Describe how the forces acting on the ball cause its acceleration to be centripetal.

....

....

....

.... [2]

(c) The ball moves in a circle of radius 0.15 m.

Show that the speed of the ball is 1.4 m s^{-1}.

[3]

(d) Calculate the angular speed ω of the ball.

ω = ... rad s^{-1} [2]

(e) The speed of the ball is increased.

Explain why the radius of the circular path of the ball increases.

....

....

.... [1]

[Total: 9]