A-LEVEL PHYSICS • EXERCISE SHEET

6.1 Stress and strain

Name: ______________ Class: __________ Date: ____________ **Total: 27 marks**

KEY WORDS young modulus 杨氏模量 • spring constant 劲度系数 • stress 应力 • extension 伸长量 • strain 应变

Objective

Build the skills to answer exam questions on **stress and strain** — Hooke's law, the spring constant, and the Young modulus.

You must be able to:

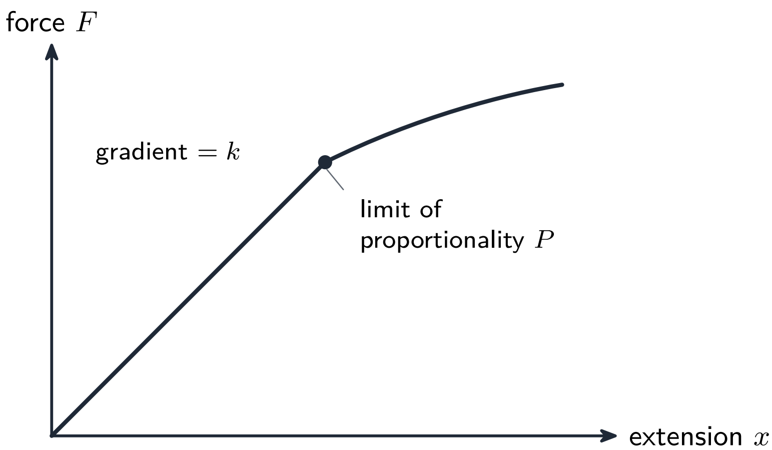
- use **Hooke's law** $F = kx$ and find k from a graph
- define and use **stress**, **strain** and the **Young modulus**
- describe the experiment to measure the **Young modulus** of a wire

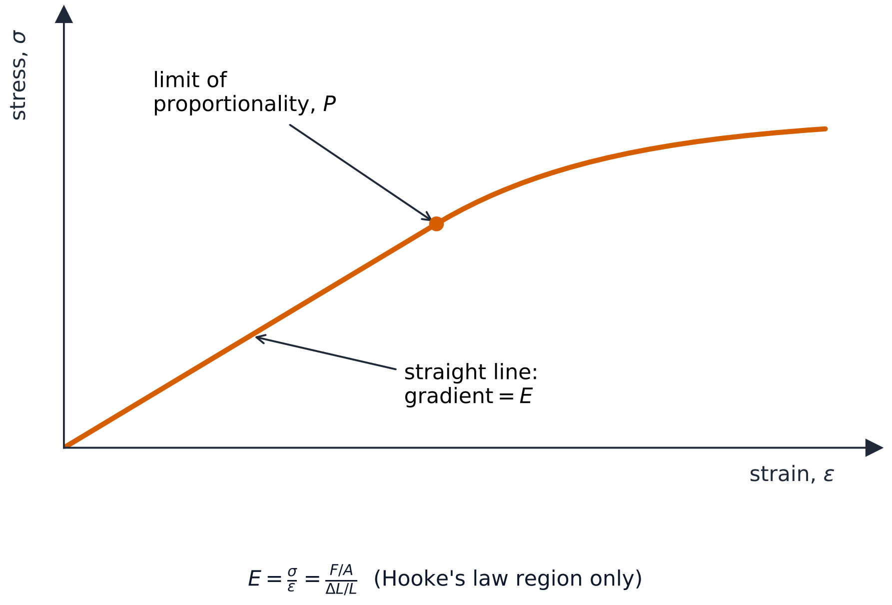
1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Hooke's law and spring constant

For small loads, **extension is proportional to load**: $F = kx$, so the **spring constant** $k = F/x$ (unit N m^{-1}). On a force–extension graph the **gradient is** k in the straight region, up to the **limit of proportionality**:





The stress-strain graph for a material

Worked example. A spring extends 4.0 cm under a 2.0 N load: $k = F/x = 2.0/0.040 = 50\ \text{N m}^{-1}$.

Stress, strain, Young modulus

$$\text{stress } \sigma = \frac{F}{A}\ (\text{Pa}), \qquad \text{strain } \varepsilon = \frac{x}{L}\ (\text{no unit}), \qquad E = \frac{\sigma}{\varepsilon}\ (\text{Pa}).$$

Young modulus experiment (wire): measure the original length L and diameter ($\rightarrow$ area A); add known loads; measure the extension x (e.g. with a marker and ruler/travelling microscope); plot stress against strain; E is the gradient of the straight part.

2 Practice

Now apply the methods above.

2.1 State **Hooke's law**. [1]

2.2 A spring extends 4.0 cm when a 2.0 N load is hung from it. Find the **spring constant**. [2]

2.3 Define **stress** and **strain**. [2]

2.4 State the unit of the **Young modulus**. [1]

3 Exam-style questions

3.1 In the straight region, the **gradient** of a force–extension graph gives the: [1]

- **A** Young modulus
- **B** spring constant
- **C** stress
- **D** strain

3.2 A wire of length 2.0 m and cross-sectional area 1.0×10^{-6} m^2 stretches by 1.0 mm under a 100 N load. Calculate (a) the stress, (b) the strain, (c) the Young modulus. [4]

3.3 Describe an experiment to measure the **Young modulus** of a metal wire. [4]

3.4 Explain what is meant by the **limit of proportionality**. [1]

3.5 A wire has length L, cross-sectional area A, Young modulus E and resistivity ρ.

(a) **Define** the **Young modulus** of a material. [1]

(b) **State** an expression, in terms of some or all of L, A, E and ρ, for the resistance R_0 of the wire. [1]

(c) **Show that** the spring constant k_0 of the wire is given by $k_0 = \dfrac{EA}{L}$. [2]

(d) The wire is now stretched by a steadily increasing force F, staying within its limit of proportionality. In the space below, **sketch** two graphs: the variation with F of the **resistance** R of the wire, and the variation with F of its **spring constant** k. **Label** each graph and its axes. [3]

(e) A copper wire of diameter 1.6 mm has a resistance of 0.034 Ω. The resistivity of copper is $1.7 \times 10^{-8}\ \Omega\,\mathrm{m}$. **Show that** the length of the wire is about 3.8 m. [2]

(f) The Young modulus of copper is 1.3×10^{11} Pa. **Determine** the spring constant of this wire. [2]

Solutions

2.1 The **extension is proportional to the load** (up to the limit of proportionality).

2.2 $k = F/x = 2.0/0.040 = 50$ N m^{-1}.

2.3 **Stress** = force per unit cross-sectional area (F/A); **strain** = extension per unit original length (x/L).

2.4 **Pa** (N m^{-2}).

3.1 **B** — spring constant.

3.2 (a) $\sigma = F/A = 100/(1.0 \times 10^{-6}) = 1.0 \times 10^{8}$ Pa. (b) $\varepsilon = x/L = (1.0 \times 10^{-3})/2.0 = 5.0 \times 10^{-4}$. (c) $E = \sigma/\varepsilon = (1.0 \times 10^{8})/(5.0 \times 10^{-4}) = 2.0 \times 10^{11}$ Pa.

3.3 Any four: measure original length L and diameter ($\rightarrow$ area A); hang the wire and add known loads; measure the extension with a marker against a fixed scale; plot stress against strain and find E from the gradient of the straight part.

3.4 The point beyond which **extension is no longer proportional to load** (the F–x line stops being straight).

3.5 (a) The **ratio of stress to strain** (for a material below its limit of proportionality).
(b) $R_0 = \dfrac{\rho L}{A}$.
(c) The spring constant is $k = \dfrac{F}{x}$, and the Young modulus is $E = \dfrac{\text{stress}}{\text{strain}} = \dfrac{F/A}{x/L} = \dfrac{FL}{Ax}$.
Rearranging, $E = \dfrac{F}{x} \cdot \dfrac{L}{A} = \dfrac{k_0 L}{A}$, so $k_0 = \dfrac{EA}{L}$.
(d) **Resistance:** a straight line of **positive** gradient starting at $(0, R_0)$ — stretching makes the wire longer and thinner, so R rises. **Spring constant:** a **horizontal** line starting at $(0, k_0)$ — within the limit of proportionality k is a constant of the wire. Both axes labelled, each line labelled.
(e) $A = \pi r^2 = \pi(0.80 \times 10^{-3})^2 = 2.01 \times 10^{-6}$ m^2; $L = \dfrac{R_0 A}{\rho} = \dfrac{0.034 \times 2.01 \times 10^{-6}}{1.7 \times 10^{-8}} = 3.8$ m. (The **radius** is half the diameter — the commonest slip here.)
(f) $k_0 = \dfrac{EA}{L} = \dfrac{1.3 \times 10^{11} \times 2.01 \times 10^{-6}}{3.8} = 6.9 \times 10^{4}$ N m^{-1}.