

5.2 Gravitational potential energy and kinetic energy

Name: _____ Class: _____ Date: _____

Total: 34 marks

KEY WORDS energy 能量 • kinetic energy 动能 • gravitational potential energy 重力势能 • mass 质量
• work 功

Objective

Build the skills to answer exam questions on **gravitational potential energy and kinetic energy** — the formulae, their derivations, and energy exchange.

You must be able to:

- use $\Delta E_P = mg\Delta h$ and derive it from $W = Fs$
- use $E_K = \frac{1}{2}mv^2$ and derive it from the equations of motion
- solve problems where GPE and KE exchange

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ GPE change

Lifting mass m slowly through height Δh needs an upward force mg , so the work done (= GPE gained) is

$$\Delta E_P = W = (mg)(\Delta h) = mg\Delta h.$$

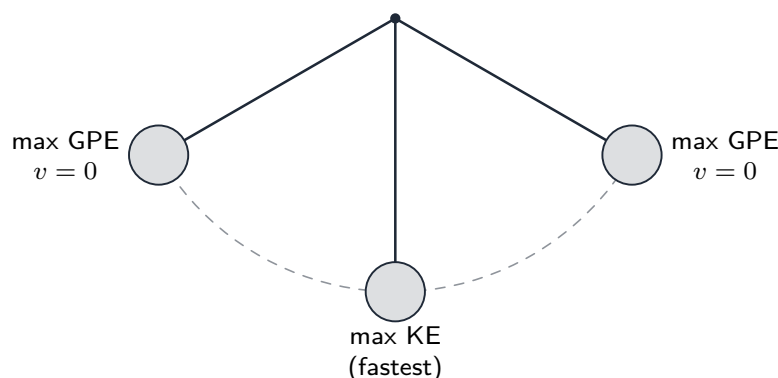
■ Kinetic energy

A force F accelerates a mass from rest to speed v over distance s . With $u = 0$, $v^2 = 2as$, so $s = v^2/2a$, and

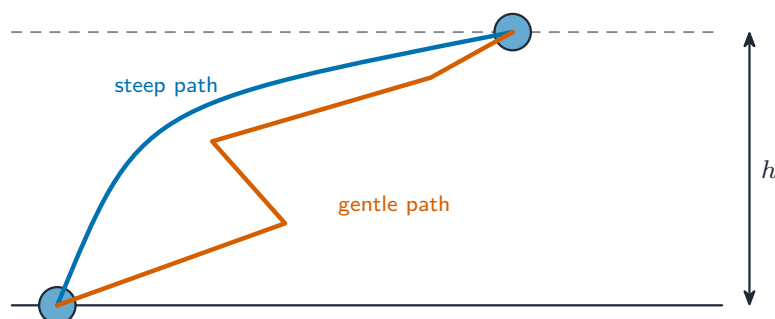
$$E_K = W = Fs = (ma)\frac{v^2}{2a} = \frac{1}{2}mv^2.$$

■ Energy exchange (pendulum)

A pendulum bob trades GPE and KE: **max GPE** (and $v = 0$) at the ends, **max KE** at the bottom:



same $\Delta E_P = mgh$ — only the height h matters



GPE depends only on height, not the path

Energy is conserved: $mgh = \frac{1}{2}mv^2$, so the bob's speed at the bottom is $v = \sqrt{2gh}$.

■ Energy accounting in a chain of transfers

The hardest energy questions are not one formula: they follow the energy through **several** stores and ask you to keep a running total. A falling block that lands on a spring is the standard case:

$$E_{\text{elastic, max}} = E_{\text{K on contact}} + \Delta E_P \text{ during compression}$$

The second term is easy to forget — the block keeps falling *while* it compresses the spring, so it loses **more** potential energy after contact.

Two more results the exam builds on top:

- the **area under a force-compression graph** is the elastic potential energy, so for a spring obeying Hooke's law $E = \frac{1}{2}F_0x_0$, which gives $F_0 = \frac{2E}{x_0}$;
- at maximum compression the spring pushes up with F_0 while the weight still pulls down, so the **resultant** force is $F_0 - mg$ and the acceleration is $\frac{F_0 - mg}{m}$ — large, upward, and the moment the block starts back up.

2 Practice

Now apply the methods above.

2.1 Write the formula for **GPE change** and for **kinetic energy**. [2]

2.2 Find the **kinetic energy** of a 1200 kg car moving at 15 m s^{-1} . [2]

2.3 Find the **GPE gained** by a 2.0 kg book lifted 1.5 m ($g = 9.81 \text{ m s}^{-2}$). [2]

2.4 At the **bottom** of a pendulum swing, which energy is at a maximum? [1]

3 Exam-style questions

3.1 An object's speed **doubles**. Its kinetic energy: [1]

- **A** doubles
 - **B** halves
 - **C** becomes four times larger
 - **D** stays the same
-

3.2 Derive $E_K = \frac{1}{2}mv^2$ using the equations of motion. [3]

3.3 A 0.50 kg ball is dropped from a height of 1.8 m. Calculate its speed just before it

hits the ground (ignore air resistance).

[3]

3.4 A pendulum bob is released from a point 0.20 m above its lowest point. Calculate its **maximum speed**.

[2]

3.5 A 0.20 kg ball is dropped from a height of 1.6 m and rebounds to a height of 1.2 m. Calculate the energy **transferred to other forms** during the bounce.

[3]

3.6 A vertical spring stands on a horizontal surface with its lower end fixed. Its mass is negligible. A block of mass 4.0 kg falls onto the spring and is brought to rest as the spring compresses. The block has kinetic energy 72 J at the instant it touches the spring.

(a) **Calculate** the speed of the block as it touches the spring.

[2]

(b) While the spring compresses, the gravitational potential energy of the block falls by a further 12 J. **Show that** the maximum compression x_0 of the spring is about 0.31 m.

[2]

(c) All the energy lost by the block becomes elastic potential energy in the spring. **De-**

termine the maximum elastic potential energy stored. [1]

(d) The spring obeys Hooke's law, so the force it exerts is proportional to the compression. **Show that** the maximum force F_0 the spring exerts on the block is about 540 N. [2]

(e) At maximum compression, **determine**

(i) the resultant force on the block, [2]

(ii) the acceleration of the block. [2]

(f) **Sketch** a graph to show how the **kinetic energy** of the block varies with the compression of the spring, from the instant of contact to maximum compression. **Label** both axes. [2]

(g) The same block is dropped from a greater height onto the same spring. **Explain**

why the maximum compression is greater, and why the maximum acceleration is greater too. [2]

Solutions

2.1 $\Delta E_P = mg\Delta h$; $E_K = \frac{1}{2}mv^2$.

2.2 $E_K = \frac{1}{2}(1200)(15)^2 = 1.35 \times 10^5 \text{ J}$.

2.3 $\Delta E_P = mg\Delta h = 2.0 \times 9.81 \times 1.5 = 29 \text{ J}$.

2.4 Kinetic energy.

3.1 C — $\text{KE} \propto v^2$, so doubling v makes KE four times larger.

3.2 Work done $W = Fs = mas$; with $u = 0$, $v^2 = 2as \Rightarrow s = v^2/2a$; $W = ma \cdot \frac{v^2}{2a} = \frac{1}{2}mv^2 = E_K$.

3.3 $\frac{1}{2}mv^2 = mgh$; $v = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 1.8} = 5.9 \text{ m s}^{-1}$.

3.4 $v = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 0.20} = 2.0 \text{ m s}^{-1}$.

3.5 GPE before $= mgh_1 = 0.20 \times 9.81 \times 1.6 = 3.1 \text{ J}$; GPE after $= mgh_2 = 0.20 \times 9.81 \times 1.2 = 2.4 \text{ J}$; energy transferred $= 3.1 - 2.4 = 0.79 \text{ J}$.

3.6 (a) $E_K = \frac{1}{2}mv^2$, so $72 = \frac{1}{2}(4.0)v^2$; $v = \sqrt{36} = 6.0 \text{ m s}^{-1}$.

(b) The block falls a further x_0 while compressing the spring, so $\Delta E_P = mgx_0$; $x_0 = \frac{12}{4.0 \times 9.81} = 0.31 \text{ m}$.

(c) All of it goes into the spring: $72 + 12 = 84 \text{ J}$.

(d) The area under the force-compression graph is the energy stored, and for a straight line that area is a triangle: $E = \frac{1}{2}F_0x_0$; $F_0 = \frac{2 \times 84}{0.306} = 5.5 \times 10^2 \text{ N}$.

(e)(i) The spring pushes **up** with F_0 and the weight pulls **down**: weight $= 4.0 \times 9.81 = 39 \text{ N}$; resultant $= 549 - 39 = 5.1 \times 10^2 \text{ N}$, upwards.

(e)(ii) $a = \frac{F}{m} = \frac{510}{4.0} = 1.3 \times 10^2 \text{ m s}^{-2}$, upwards — about 13 times g .

(f) Axes labelled *kinetic energy / J* against *compression / m*; a curve starting at 72 J when the compression is zero and falling to zero at $x_0 = 0.31 \text{ m}$, getting steeper as the spring force grows.

(g) A greater drop height gives the block **more kinetic energy** at contact, so more energy must be stored in the spring; since $E = \frac{1}{2}kx^2$, a larger E needs a larger compression x_0 .

The spring force is proportional to the compression, so F_0 is larger too, and $a = \frac{F_0 - mg}{m}$ therefore rises with it.