

4.3 Density and pressure

Name: _____ Class: _____ Date: _____

Total: 31 marks

KEY WORDS density 密度 • pressure 压强 • upthrust 浮力 • hydrostatic pressure 流体静压强 • atmospheric pressure 大气压强

Objective

Build the skills to answer exam questions on **density and pressure** — including hydrostatic pressure and upthrust.

You must be able to:

- use **density** $\rho = m/V$ and **pressure** $p = F/A$
- derive and use **hydrostatic pressure** $\Delta p = \rho g \Delta h$
- find **upthrust** with $F = \rho g V$ (Archimedes' principle)
- get a density from **measured** side lengths, and its **percentage uncertainty**
- decide whether an object **floats**, and how much of it sits below the surface

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Density and pressure

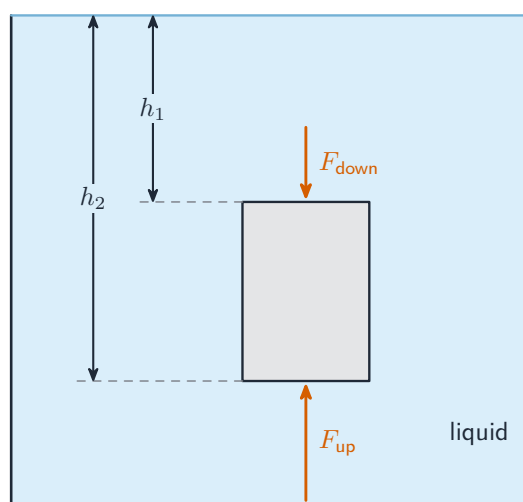
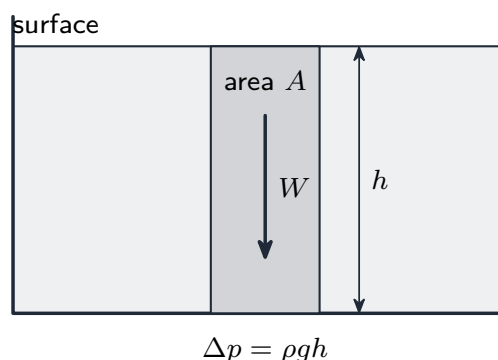
$$\rho = \frac{m}{V} \text{ (kg m}^{-3}\text{)}, \quad p = \frac{F}{A} \text{ (Pa = N m}^{-2}\text{)}.$$

Water has $\rho = 1000 \text{ kg m}^{-3}$.

■ Hydrostatic pressure

A column of liquid (area A , depth h) has weight $W = \rho Ahg$ pressing on its base, so the **extra** pressure is

$$\Delta p = \frac{W}{A} = \rho gh.$$



Upthrust arises from the pressure difference with depth

It depends only on **depth and density** — not on the container's shape. (For total pressure, add atmospheric pressure $\approx 1.0 \times 10^5$ Pa.)

■ Upthrust

A submerged object feels a larger pressure on its **bottom** than its top — this pressure difference gives an upward **upthrust**:

$$F = \rho g V,$$

where V is the volume of fluid displaced (Archimedes' principle).

■ Density from measurements, with its uncertainty

An exam density is rarely handed to you: it comes from a mass and three measured sides, each with an uncertainty.

Worked example. $m = (0.243 \pm 0.001)$ kg; $x = (5.41 \pm 0.01)$ cm, $y = (11.09 \pm 0.01)$ cm, $z = (1.62 \pm 0.01)$ cm.

$$V = xyz = 5.41 \times 11.09 \times 1.62 = 97.2 \text{ cm}^3 = 9.72 \times 10^{-5} \text{ m}^3$$

$$\rho = \frac{m}{V} = \frac{0.243}{9.72 \times 10^{-5}} = 2500 \text{ kg m}^{-3}$$

For the uncertainty, add the **percentage** uncertainties of everything multiplied or divided:

$$\frac{0.001}{0.243} + \frac{0.01}{5.41} + \frac{0.01}{11.09} + \frac{0.01}{1.62} = 0.41 + 0.18 + 0.09 + 0.62 = 1.3\%$$

The **smallest** side carries the largest percentage — that is the measurement worth improving, and examiners ask exactly that.

■ Floating

An object floats when the upthrust can balance its weight **before** it is fully under:

$$\rho_{\text{object}} V_{\text{total}} g = \rho_{\text{fluid}} V_{\text{submerged}} g \quad \Rightarrow \quad \frac{V_{\text{submerged}}}{V_{\text{total}}} = \frac{\rho_{\text{object}}}{\rho_{\text{fluid}}}$$

So ice (920 kg m^{-3}) floats in water with 92% of its volume below the surface. If $\rho_{\text{object}} > \rho_{\text{fluid}}$ the fraction would exceed 1, which is the algebra telling you it sinks.

2 Practice

Now apply the methods above.

2.1 Define **density** and give its SI unit. [1]

2.2 A block has mass 2400 kg and volume 0.30 m^3 . Find its **density**. [2]

2.3 A force of 200 N acts on an area of 0.50 m^2 . Find the **pressure**. [2]

2.4 State what causes the **upthrust** on a submerged object. [1]

3 Exam-style questions

3.1 The extra pressure at the bottom of a liquid column depends on: [1]

- **A** the cross-sectional area only
 - **B** the depth and the density
 - **C** the shape of the container
 - **D** the total volume of liquid
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3.2 Calculate the extra pressure due to the water at the bottom of a pool 2.5 m deep. ($\rho = 1000 \text{ kg m}^{-3}$, $g = 9.81 \text{ m s}^{-2}$.) [2]

3.3 A block of volume $2.0 \times 10^{-3} \text{ m}^3$ is fully submerged in water ($\rho = 1000 \text{ kg m}^{-3}$). Calculate the **upthrust** on it. [2]

3.4 Derive the hydrostatic pressure equation $\Delta p = \rho gh$ from the definitions of **pressure** and **density**. [3]

3.5 A student measures a rectangular metal block to find the density of the metal. The results are

quantity	measurement
mass m	$(0.512 \pm 0.001) \text{ kg}$
length x	$(8.02 \pm 0.01) \text{ cm}$
width y	$(4.15 \pm 0.01) \text{ cm}$
height z	$(2.05 \pm 0.01) \text{ cm}$

- (a) **Define** density. [1]

- (b) **Determine** the density of the metal. [3]

- (c) **Determine** the percentage uncertainty in the density. [3]

- (d) **State**, with a reason, which single measurement the student should improve first. [2]

- (e) The block is lowered into water ($\rho = 1000 \text{ kg m}^{-3}$) until it is fully submerged. **Show that** the upthrust on it is about 0.67 N, and **explain** why the block still sinks. [3]

- (f) A block of wood of density 650 kg m^{-3} floats in the same water. **Determine** the

fraction of its volume that lies **below** the surface.

[2]

(g) A swimming pool is 25 m long, 10 m wide and 2.0 m deep. **Estimate** the force that the water exerts on the bottom of the pool, and state one assumption you make. [3]

Solutions

2.1 Density is **mass per unit volume**, unit kg m^{-3} .

2.2 $\rho = m/V = 2400/0.30 = 8000 \text{ kg m}^{-3}$.

2.3 $p = F/A = 200/0.50 = 400 \text{ Pa}$.

2.4 The **difference in pressure** between the bottom and top of the object (the bottom is deeper, so at higher pressure).

3.1 B — the depth and the density.

3.2 $\Delta p = \rho gh = 1000 \times 9.81 \times 2.5 = 2.5 \times 10^4 \text{ Pa}$.

3.3 $F = \rho gV = 1000 \times 9.81 \times 2.0 \times 10^{-3} = 19.6 \text{ N}$.

3.4 The column's mass $m = \rho V = \rho Ah$; its weight $W = mg = \rho Ahg$ presses on area A ; so $\Delta p = W/A = \rho gh$.

3.5 (a) Density is **mass per unit volume**.

(b) $V = 8.02 \times 4.15 \times 2.05 = 68.2 \text{ cm}^3 = 6.82 \times 10^{-5} \text{ m}^3$; $\rho = \frac{0.512}{6.82 \times 10^{-5}} = 7.5 \times 10^3 \text{ kg m}^{-3}$.

(c) $\frac{0.001}{0.512} = 0.20\%$; $\frac{0.01}{8.02} = 0.12\%$; $\frac{0.01}{4.15} = 0.24\%$; $\frac{0.01}{2.05} = 0.49\%$; total = $0.20 + 0.12 + 0.24 + 0.49 = 1.1\%$.

(d) The **height** z — it is the smallest length, so the same $\pm 0.01 \text{ cm}$ is the largest percentage of it and it dominates the total.

(e) $F = \rho_{\text{water}} gV = 1000 \times 9.81 \times 6.82 \times 10^{-5} = 0.67 \text{ N}$. The block's own weight is $mg = 0.512 \times 9.81 = 5.0 \text{ N}$, far greater than the upthrust, so the resultant force is downwards and it sinks — its density is over seven times that of water.

(f) Floating means upthrust = weight, so $\rho_{\text{water}} V_{\text{sub}} g = \rho_{\text{wood}} V_{\text{total}} g$; $\frac{V_{\text{sub}}}{V_{\text{total}}} = \frac{650}{1000} = 0.65$, i.e. 65% below the surface.

(g) Pressure due to the water at the bottom: $\Delta p = \rho gh = 1000 \times 9.81 \times 2.0 = 1.96 \times 10^4 \text{ Pa}$; area = $25 \times 10 = 250 \text{ m}^2$, so $F = pA = 1.96 \times 10^4 \times 250 = 4.9 \times 10^6 \text{ N}$. Assumption: the depth is the same everywhere (a flat bottom), and atmospheric pressure is not included — adding it would roughly double the force.