4.2 Equilibrium of forces

Name: ______________ Class: __________ Date: ____________ **Total: 30 marks**

KEY WORDS equilibrium 平衡 • vector triangle 矢量三角形 • force 力 • moment 力矩 • principle of moments 力矩

Objective

Build the skills to answer exam questions on **equilibrium 平衡** — the principle of moments, the two conditions for equilibrium, and the vector triangle.

You must be able to:

- state and apply the **principle of moments**
- use the **two conditions** for equilibrium (no resultant force, no resultant moment)
- use a closed **vector triangle** for three coplanar forces, with numbers
- **draw** labelled force arrows on a diagram before calculating anything
- take moments about a **hinge** and find the force the hinge itself exerts

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

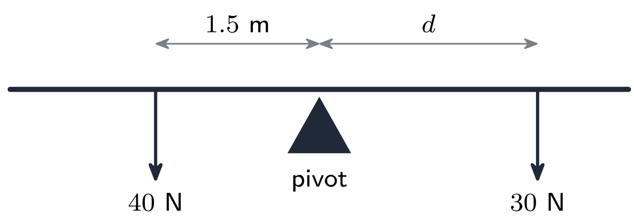
Conditions for equilibrium

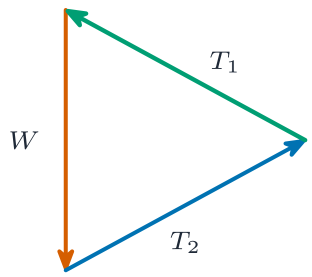
A body is in **equilibrium** when **both**:

1. the **resultant force** is zero, and
2. the **resultant moment** (about any point) is zero.

Principle of moments

For a body not turning, the **total clockwise moment** = **total anticlockwise moment** about any point.





A closed vector triangle shows equilibrium

Worked example. Beam pivoted at its centre (so its weight gives no moment), 40 N at 1.5 m on the left:

$$40 \times 1.5 = 30 \times d \Rightarrow d = \frac{60}{30} = 2.0 \text{ m}.$$

Tip: take moments about the point where an **unknown** force acts, so it drops out.

■ Vector triangle

Three coplanar forces in equilibrium form a **closed triangle** (tip-to-tail). Solve it with the sine/cosine rule, or resolve into components and set $\sum F_x = 0$, $\sum F_y = 0$.

Worked example. A 250 N sign hangs from a horizontal wire and a wire at 40° to the vertical. Resolving the sloping tension T:

$$\text{vertical: } T\cos 40° = 250 \Rightarrow T = \frac{250}{0.766} = 326 \text{ N}$$

$$\text{horizontal: } T\sin 40° = F \Rightarrow F = 326 \times 0.643 = 210 \text{ N}$$

The vertical equation comes first because only **one** unknown appears in it. A closed triangle drawn to scale gives the same two answers.

■ Forces on a hinged beam

A beam hinged at one end and held up somewhere else has **three** forces on it, and the hinge one is the awkward one — so:

1. **Draw the arrows first**, labelled: the weight at the **centre of gravity**, the support force, the load.
2. **Take moments about the hinge**, because its own force then has zero moment and drops out.
3. Get the hinge force **last**, from vertical equilibrium — and be ready for it to point **down**. If the support is between the hinge and the load, the hinge must pull the beam down to stop it tipping.

2 Practice

Now apply the methods above.

2.1 State the **two** conditions for a body to be in **equilibrium**. [2]

2.2 State the **principle of moments**. [1]

2.3 A 40 N weight hangs 1.5 m to the left of a pivot. How far to the **right** must a 30 N weight hang to balance it? [2]

2.4 State what a **closed vector triangle** tells you about three forces. [1]

3 Exam-style questions

3.1 Three coplanar forces act on an object that is in equilibrium. Which statement **must** be true? [1]

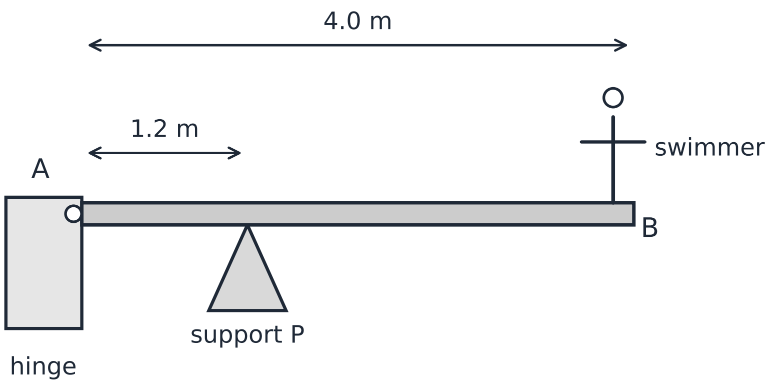
- **A** The three forces are equal in magnitude.
- **B** Drawn tip to tail, the three forces form a closed triangle.
- **C** The three forces all act through the same point and are parallel.
- **D** The resultant of two of the forces is equal in both magnitude and direction to the third.

3.2 A uniform beam of length 4.0 m is pivoted at its centre. A 40 N weight hangs 1.5 m from the pivot. Calculate the distance on the other side at which a 30 N weight balances the beam. [3]

3.3 A uniform beam of weight 60 N and length 4.0 m rests on supports **A** (left end) and **B** (right end). A 40 N load sits 1.0 m from A. **Show that** the upward force on support **B** is 40 N. [3]

3.4 Explain why, when balancing a **heavy uniform rod**, you must include its **weight** in the moments calculation. [2]

3.5 A uniform diving board AB has length 4.0 m and weight 800 N. It is hinged to a wall at end A and rests on a support P a distance 1.2 m from A. A swimmer of weight 620 N stands at end B, and the board is in equilibrium.



(a) **State** the principle of moments. [2]

(b) On the diagram above, **draw** arrows showing the **weight of the board**, the **force from the support P** and the **weight of the swimmer**. **Label** each arrow. [3]

(c) By taking moments about A, **determine** the force that the support P exerts on the board. [3]

(d) **Determine** the magnitude and the direction of the force that the hinge at A exerts on the board. [3]

(e) **Explain** why taking moments about A is a better choice than taking them about P. [1]

(f) The swimmer now walks slowly back towards A. **Give** the distance from A at which she must stand for the hinge to exert **no** force on the board. [3]

Solutions

2.1 The **resultant force** is zero **and** the **resultant moment** (about any point) is zero.

2.2 For a body in equilibrium, the **total clockwise moment** about any point equals the **total anticlockwise moment** about that point.

2.3 $40 \times 1.5 = 30 \times d \Rightarrow d = 2.0$ m.

2.4 The three forces are **in equilibrium** (their resultant is zero).

3.1 **B** — in equilibrium the three forces have zero resultant, so tip to tail they close into a triangle. (D is the classic trap: the resultant of two must be equal and **opposite** to the third, not equal in direction. A and C need not hold at all.)

3.2 Beam's weight acts at the pivot (no moment); $40 \times 1.5 = 30 \times d$; $d = 60/30 = 2.0$ m.

3.3 Take moments about **A**: $B \times 4.0 = 60 \times 2.0 + 40 \times 1.0 = 120 + 40 = 160$; $B = 160/4.0 = 40$ N.

3.4 The rod's weight acts at its **centre of gravity**, at a distance from the pivot, so it contributes a **moment** that must be included or the balance equation is wrong.

3.5 (a) For a body in equilibrium, the sum of the **clockwise moments** about any point equals the sum of the **anticlockwise moments** about that same point.
(b) An arrow **down** at the middle of the board (2.0 m from A) labelled *weight of board / 800 N*; an arrow **up** at P labelled *support force*; an arrow **down** at B labelled *weight of swimmer / 620 N.*
(c) Moments about A: the hinge force has no moment there. Clockwise $= 800 \times 2.0 + 620 \times 4.0 = 1600 + 2480 = 4080$ N m; anticlockwise $= P \times 1.2$; $P = \dfrac{4080}{1.2} = 3400$ N.
(d) Vertical equilibrium: up = down, so $3400 +$ (hinge force up) $= 800 + 620$. That gives -1980 N, so the hinge force is 1980 N directed **downwards** — the hinge has to hold the board **down**, because the support is between A and the load.
(e) The force at the hinge is unknown, and moments taken **about** A give it zero moment, so it disappears from the equation.
(f) With no hinge force the board balances on P alone, so take moments about **P**. The board's weight acts $2.0 - 1.2 = 0.8$ m to the right of P, giving a clockwise moment $800 \times 0.8 = 640$ N m. The swimmer must therefore be on the **other** side of P, a distance $(1.2-x)$ from it: $620(1.2-x) = 640$, so $1.2-x = 1.03$ and $x = 0.17$ m from A. (Anywhere beyond P and both weights turn the board the same way, which no position can balance.)