

3.3 Linear momentum and its conservation

Name: _____ Class: _____ Date: _____

Total: 34 marks

KEY WORDS conservation of momentum 动量守恒 • elastic collision 弹性碰撞 • relative speed 相对速率
 • inelastic collision 非弹性碰撞 • collision 碰撞

Objective

Build the skills to answer exam questions on **conservation of momentum** 动量守恒 — collisions, explosions, and elastic vs inelastic interactions.

You must be able to:

- state and apply **conservation of momentum** (using signed velocities)
- distinguish **elastic** and **inelastic** collisions
- use the elastic test: relative speed of **approach** = relative speed of **separation**
- **calculate** the percentage of kinetic energy transferred away in a collision
- apply momentum conservation to a **nuclear decay**, and say what else is conserved

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

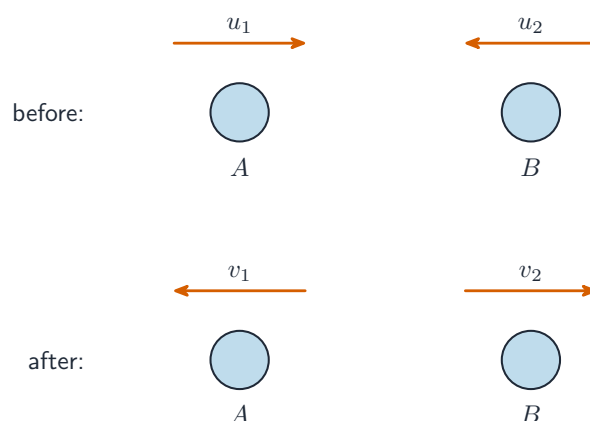
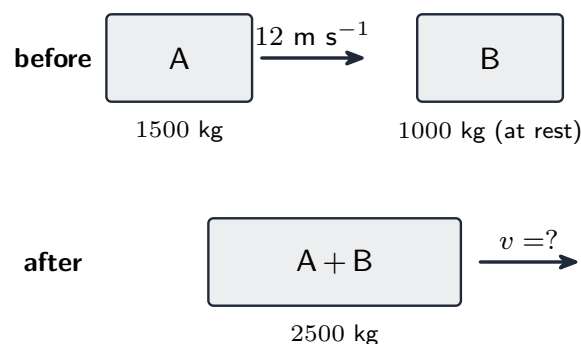
■ The principle

With **no resultant external force**, total momentum is **constant**. Always write momenta with **signs** (positive in one direction).

Explosion/recoil. A 2.0 kg and a 3.0 kg trolley spring apart from rest. Total momentum stays zero:

$$0 = (2.0)(6.0) + (3.0)(-v) \Rightarrow v = 4.0 \text{ m s}^{-1} \text{ (opposite way).}$$

■ Inelastic collision (objects stick)



Momentum is conserved in a head-on collision

Momentum conserved, $v_1 = v_2 = v$:

$$1500(12) + 1000(0) = (2500)v \Rightarrow v = \frac{18\,000}{2500} = 7.2 \text{ m s}^{-1}.$$

■ Elastic vs inelastic

- **Every** collision conserves **momentum**.
- An **elastic** collision **also** conserves **kinetic energy**; test: $u_1 - u_2 = -(v_1 - v_2)$ (relative speed of approach = of separation).
- An **inelastic** collision loses KE (to heat/sound/deformation); if they stick, it is perfectly inelastic.

■ How much kinetic energy was lost?

“Elastic or not” is usually asked as a **number**. Work out the kinetic energy on each side and compare:

$$\% \text{ transferred away} = \frac{E_{K,\text{before}} - E_{K,\text{after}}}{E_{K,\text{before}}} \times 100$$

Worked example (the two cars above). Before: $\frac{1}{2}(1500)(12)^2 = 1.08 \times 10^5 \text{ J}$. After: $\frac{1}{2}(2500)(7.2)^2 = 6.48 \times 10^4 \text{ J}$. So $\frac{1.08 \times 10^5 - 6.48 \times 10^4}{1.08 \times 10^5} \times 100 = 40\%$ is transferred

to other forms. **Momentum is unchanged** all the same — that is the point of the question.

■ **A decay is an explosion**

A stationary nucleus that decays is a two-body explosion, so the total momentum stays **zero**:

$$m_1v_1 = m_2v_2$$

The masses can be left in **atomic mass units** — they appear on both sides, so they cancel out of the ratio. Momentum, **charge (proton number)**, **nucleon number** and **total energy** are all conserved; kinetic energy is **not**, because energy is released from the nucleus.

2 Practice

Now apply the methods above.

2.1 State the **principle of conservation of momentum**. [1]

2.2 State what is conserved in (a) an **elastic** and (b) an **inelastic** collision. [2]

2.3 Two trolleys, 2.0 kg and 3.0 kg, are pushed apart from rest by a spring. The 2.0 kg trolley moves off at 6.0 m s^{-1} . Find the speed of the 3.0 kg trolley. [2]

2.4 State the test (using relative speeds) for an **elastic** collision. [1]

3 Exam-style questions

3.1 In an **inelastic** collision, which quantity is conserved? [1]

- **A** kinetic energy only
 - **B** momentum only
 - **C** both momentum and kinetic energy
 - **D** neither
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3.2 A 1500 kg car moving at 12 m s^{-1} hits a stationary 1000 kg car and they lock together. Calculate their **common velocity** just after the collision. [3]

3.3 A 0.50 kg ball moving at 4.0 m s^{-1} collides **elastically** and head-on with a stationary 0.50 kg ball. State the velocity of **each** ball after the collision. [2]

3.4 Explain why the **kinetic energy** can decrease in a collision while the **momentum** does not. [2]

3.5 On a horizontal frictionless track, trolley A of mass 1.6 kg moves to the right at 4.5 m s^{-1} . Ahead of it, trolley B of mass 0.90 kg moves to the right at 1.5 m s^{-1} . A catches B, and the two move off together.

(a) **State** the principle of conservation of momentum. [2]

- (b) **Determine** the common velocity of the trolleys after the collision. [3]

- (c) **Calculate** the percentage of the total kinetic energy that is transferred to other forms during the collision. [3]

- (d) **State and justify** whether this collision is elastic. [2]

3.6 A radon-220 nucleus, at rest and far from anything else, breaks apart into a polonium nucleus and an α -particle. Measurements show the polonium nucleus moves off at $2.2 \times 10^5 \text{ m s}^{-1}$.

- (a) **Complete** the equation for the decay. [1]



- (b) By considering momentum, **calculate** the speed of the α -particle. [3]

- (c) **State** three quantities that are conserved as the nucleus breaks apart. [3]

(d) **Determine** the ratio $\frac{\text{kinetic energy of the } \alpha\text{-particle}}{\text{kinetic energy of the polonium nucleus}}$, and comment on which fragment carries away most of the released energy. [3]

Solutions

2.1 With **no resultant external force**, the **total momentum** of a system stays **constant**.

2.2 (a) Elastic — **momentum and kinetic energy**; (b) inelastic — **momentum only** (KE decreases).

2.3 $0 = (2.0)(6.0) + (3.0)(-v) \Rightarrow v = 12/3.0 = 4.0 \text{ m s}^{-1}$.

2.4 The **relative speed of approach** equals the **relative speed of separation**.

3.1 B — momentum only.

3.2 $1500(12) + 1000(0) = (1500 + 1000)v$; $18\,000 = 2500v$; $v = 7.2 \text{ m s}^{-1}$.

3.3 Equal masses, elastic, target at rest: the first ball **stops** (0 m s^{-1}) and the second moves off at 4.0 m s^{-1} .

3.4 In a collision the equal-and-opposite forces conserve total momentum; but kinetic energy can be transferred to **thermal/sound/deformation** energy, so total KE can decrease.

3.5 (a) For a system with **no resultant external force** (an isolated system), the **total momentum stays constant** — total momentum before = total momentum after.

(b) $p_{\text{before}} = (1.6)(4.5) + (0.90)(1.5) = 7.2 + 1.35 = 8.55 \text{ kg m s}^{-1}$; after the collision the mass is $1.6 + 0.90 = 2.5 \text{ kg}$; $v = \frac{8.55}{2.5} = 3.4 \text{ m s}^{-1}$ to the right.

(c) $E_{\text{K, before}} = \frac{1}{2}(1.6)(4.5)^2 + \frac{1}{2}(0.90)(1.5)^2 = 16.2 + 1.01 = 17.2 \text{ J}$; $E_{\text{K, after}} = \frac{1}{2}(2.5)(3.42)^2 = 14.6 \text{ J}$; $\frac{17.2 - 14.6}{17.2} \times 100 = 15\%$.

(d) **Not elastic** — kinetic energy is not conserved: about 15% of it is transferred to thermal and sound energy and to deforming the trolleys.

3.6 (a) ${}^{220}_{86}\text{Rn} \rightarrow {}^{216}_{84}\text{Po} + {}^4_2\alpha$ — nucleon number $220 = 216 + 4$ and proton number $86 = 84 + 2$.

(b) The nucleus is stationary, so the total momentum stays **zero** and the two fragments carry equal and opposite momentum: $4u \times v_\alpha = 216u \times 2.2 \times 10^5$, so $v_\alpha = \frac{216 \times 2.2 \times 10^5}{4} = 1.2 \times 10^7 \text{ m s}^{-1}$, in the opposite direction. (The mass unit u cancels.)

(c) Any three of: **momentum, total energy, charge (proton number), nucleon number**. Kinetic energy is **not** conserved — it is created from the nucleus's store of energy.

(d) The momenta are equal in magnitude, and $E_{\text{K}} = \frac{p^2}{2m}$, so for the same p the kinetic energy is **inversely proportional to the mass**: $\frac{E_\alpha}{E_{\text{Po}}} = \frac{m_{\text{Po}}}{m_\alpha} = \frac{216}{4} = 54$. The light α -particle carries away about 98% of the released energy — which is why it, not the recoiling nucleus, is what a detector sees.