

23.2 Radioactive decay

Name: _____ Class: _____ Date: _____

Total: 32 marks

KEY WORDS activity 活度 • half-life 半衰期 • spontaneous 自发 • decay constant 衰变常数 • random 随机

Objective

Build the skills to answer exam questions on **radioactive decay**.

You must be able to:

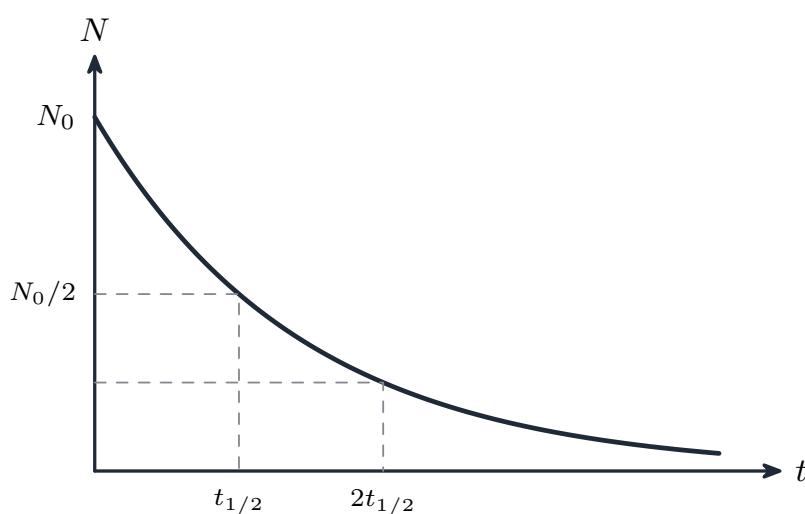
- describe decay as **spontaneous** and **random**
- use **activity** $A = \lambda N$ and $\lambda = 0.693/t_{1/2}$
- use the exponential decay $x = x_0 e^{-\lambda t}$ and **half-life**

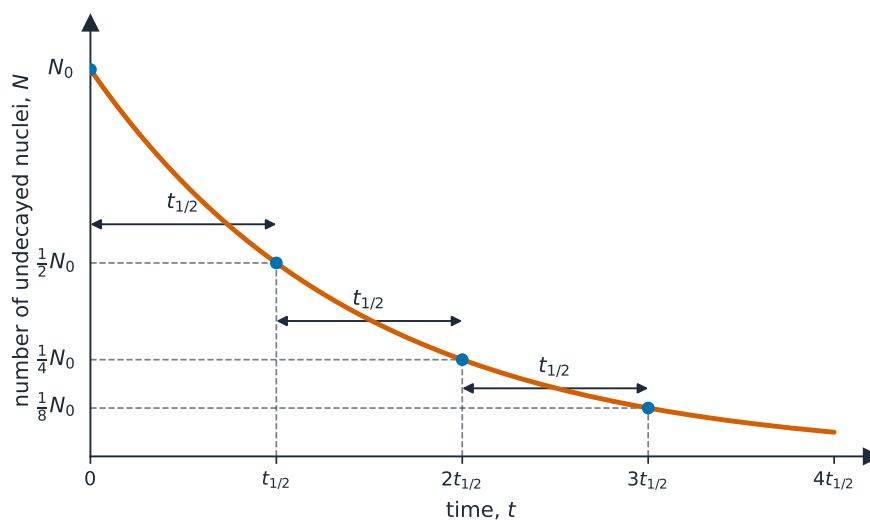
1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Random and exponential

Decay is **spontaneous** (no trigger) and **random** (the count rate fluctuates). The number of undecayed nuclei falls **exponentially**, halving every **half-life**:





$$N = N_0 e^{-\lambda t} \quad T_{1/2} = \ln 2 / \lambda$$

The exponential radioactive decay curve

$$A = \lambda N, \quad \lambda = \frac{0.693}{t_{1/2}}, \quad N = N_0 e^{-\lambda t}.$$

- **activity** A — decays per second (becquerel, Bq); **decay constant** λ — probability of decay per second.

The same exponential law applies to the **activity** and the **count rate**: $A = A_0 e^{-\lambda t}$.

■ Finding the time — take logs

For a **whole number** of half-lives, just halve repeatedly (after n half-lives a fraction $(\frac{1}{2})^n$ remains). For **any** time or fraction, take the natural log of $N = N_0 e^{-\lambda t}$:

$$\ln \frac{N}{N_0} = -\lambda t \quad \Rightarrow \quad t = -\frac{1}{\lambda} \ln \frac{N}{N_0}.$$

Worked example. A source has half-life 8.0 days, so $\lambda = \frac{0.693}{8.0} = 0.0866 \text{ day}^{-1}$.

The time for its activity to fall to 20% is $t = -\frac{1}{0.0866} \ln(0.20) = 18.6 \text{ days}$.

2 Practice

Now apply the methods above.

2.1 State **two** features of radioactive decay. [2]

2.2 Define **activity** and the **decay constant**. [2]

2.3 Define **half-life**. [1]

2.4 Write the equation linking **activity**, **decay constant** and **number** of nuclei. [1]

3 Exam-style questions

3.1 The **half-life** is the time for: [1]

- **A** all the nuclei to decay
 - **B** half the (undecayed) nuclei to decay
 - **C** the activity to reach exactly zero
 - **D** the decay constant to halve
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3.2 A source has a decay constant of 0.040 s^{-1} . Calculate its **half-life**. [2]

3.3 A sample has 2.0×10^{20} undecayed nuclei and a decay constant of $1.5 \times 10^{-9} \text{ s}^{-1}$. Calculate its **activity**. [2]

3.4 A radioactive isotope has a half-life of 6.0 hours. What **fraction** of the original nuclei remains after 18 hours? [2]

3.5 A radioactive source has a half-life of 8.0 days. Calculate the **time** for its activity to fall to 20% of its initial value. [4]

3.6 Oxygen-15 is used as a tracer in medical imaging. It decays by β^+ emission with a half-life of 122 s. A freshly prepared sample contains 4.8×10^{14} nuclei.

(a) **State** what is meant by a **tracer**. [2]

(b) The decay is ${}^{15}_8\text{O} \rightarrow {}^P_Q\text{N} + {}^R_S\text{Z}$. **State** the values of P , Q , R and S , and **name** the particle Z. [3]

(c) **Define** the **activity** of a radioactive sample. [1]

(d) **Determine** the decay constant of oxygen-15. Give a **unit** with your answer. [2]

(e) **Determine** the initial activity of the sample. [2]

(f) **Determine** the activity 10 minutes after preparation. [3]

(g) **Suggest** why a tracer with a half-life of a few **days** would be unsuitable for this use. [2]

Solutions

2.1 It is **spontaneous** (unaffected by external conditions) and **random** (cannot predict which nucleus decays when).

2.2 Activity — the number of decays per unit time; **decay constant** — the probability per unit time that a nucleus decays.

2.3 The time for **half** the undecayed nuclei (or the activity) to decay.

2.4 $A = \lambda N$.

3.1 B — half the undecayed nuclei to decay.

3.2 $t_{1/2} = \frac{0.693}{\lambda} = \frac{0.693}{0.040} = 17 \text{ s}.$

3.3 $A = \lambda N = (1.5 \times 10^{-9})(2.0 \times 10^{20}) = 3.0 \times 10^{11} \text{ Bq}.$

3.4 $18 \text{ h} = 3 \text{ half-lives}; \text{fraction} = \left(\frac{1}{2}\right)^3 = \frac{1}{8}.$

3.5 $\lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{8.0} = 0.0866 \text{ day}^{-1}; t = -\frac{1}{\lambda} \ln \frac{A}{A_0} = -\frac{1}{0.0866} \ln(0.20) = 18.6 \text{ days}.$

3.6 (a) A radioactive substance introduced into a system whose progress can be followed from outside by detecting the radiation it emits.

(b) $P = 15, Q = 7, R = 0, S = +1$; Z is a **positron**.

(c) The **rate of decay** — the number of nuclei decaying per unit time.

(d) $\lambda = \frac{\ln 2}{t_{1/2}} = \frac{0.693}{122} = 5.7 \times 10^{-3} \text{ s}^{-1}.$

(e) $A = \lambda N = 5.68 \times 10^{-3} \times 4.8 \times 10^{14} = 2.7 \times 10^{12} \text{ Bq}.$

(f) 10 minutes is 600 s; $A = A_0 e^{-\lambda t} = 2.73 \times 10^{12} \times e^{-5.68 \times 10^{-3} \times 600} = 2.73 \times 10^{12} \times 0.0332 = 9.1 \times 10^{10} \text{ Bq}.$ (That is about five half-lives, so roughly 1/32 of the start.)

(g) The patient would keep receiving **ionising radiation** long after the scan is over, raising the dose and the risk of cell damage; the tracer should decay away soon after the measurement, which a half-life of minutes achieves and one of days does not.