

23.1 Mass defect and nuclear binding energy

Name: _____ Class: _____ Date: _____

Total: 27 marks

KEY WORDS energy 能量 • mass defect 质量亏损 • binding energy 结合能 • binding energy per nucleon 比结合能
• nucleon number 核子数

Objective

Build the skills to answer exam questions on **mass defect and binding energy**.

You must be able to:

- use $E = mc^2$ and balance **nuclear equations**
- define **mass defect** and **binding energy**
- use the **binding energy per nucleon** curve for fusion and fission

1 Worked examples

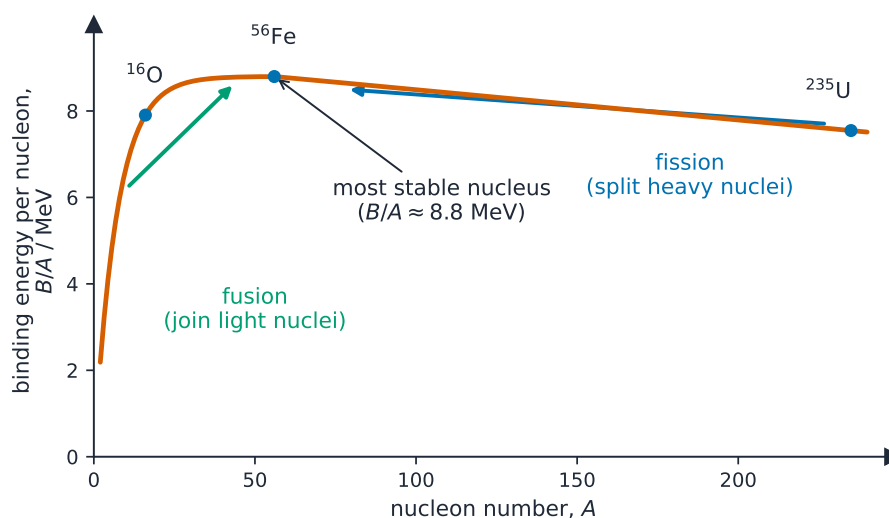
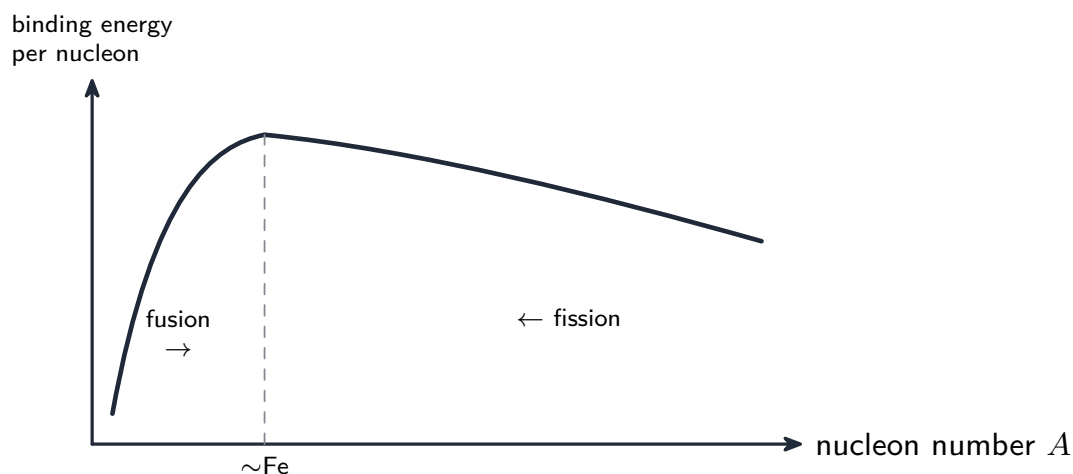
Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Mass-energy and binding energy

$$E = mc^2, \quad \Delta E = c^2 \Delta m, \quad 1 \text{ u} \approx 931 \text{ MeV}.$$

The **mass defect** is the difference between the mass of the separate nucleons and the mass of the nucleus; the **binding energy** is the energy equivalent of this ($\Delta E = c^2 \Delta m$) — the energy to split the nucleus apart.

■ Binding energy per nucleon



binding energy per nucleon peaks near ^{56}Fe

Binding energy per nucleon across the nuclides

The peak (near iron) is the **most stable**. **Fusion** of light nuclei and **fission** of heavy nuclei both move **towards the peak** — increasing binding energy per nucleon and **releasing energy**.

In nuclear equations, **nucleon number** and **charge** are conserved.

■ Energy released in a decay (from masses)

The energy released is the **mass lost** (parent mass – total product mass), converted with $1 \text{ u} \approx 931 \text{ MeV}$:

$$Q = (m_{\text{before}} - m_{\text{after}}) \times 931 \text{ MeV/u.}$$

Worked example. $^{238}_{92}\text{U} \rightarrow ^{234}_{90}\text{Th} + ^4_2\alpha$, with masses (u): U = 238.05079, Th

$= 234.04360$, $\alpha = 4.00260$. Mass lost $\Delta m = 238.05079 - (234.04360 + 4.00260) = 0.00459$ u; energy released $Q = 0.00459 \times 931 = 4.3$ MeV.

Working **backwards**: if the released energy is known, $\Delta m = Q/931$ u, which fixes a missing mass.

2 Practice

Now apply the methods above.

2.1 State **Einstein's** mass–energy equation. [1]

2.2 Define **mass defect**. [1]

2.3 Define **binding energy**. [1]

2.4 In ${}^{14}_7\text{N} + {}^4_2\text{He} \rightarrow {}^A_8\text{O} + {}^1_1\text{H}$, find the nucleon number A . [1]

3 Exam-style questions

3.1 The **most stable** nuclei have: [1]

- **A** the highest mass
 - **B** the highest binding energy per nucleon
 - **C** the lowest binding energy per nucleon
 - **D** zero binding energy
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3.2 A reaction has a mass defect of 0.030 u. Calculate the energy released. (1 u = 931 MeV.) [2]

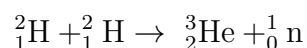
3.3 Explain how the binding-energy-per-nucleon curve shows that **both** fusion of light

nuclei **and** fission of heavy nuclei can release energy. [3]

3.4 Calculate the energy equivalent of a mass change of 1.0×10^{-29} kg. [2]

3.5 In the decay ${}^{210}_{84}\text{Po} \rightarrow {}^{206}_{82}\text{Pb} + {}^4_2\alpha$, the masses (u) are: Po = 209.98286, Pb = 205.97444, $\alpha = 4.00260$. Calculate the energy released, in MeV. (1 u = 931 MeV.) [3]

3.6 Deuterium nuclei fuse to form helium-3 in the reaction



The masses are: deuterium 2.01410 u, helium-3 3.01603 u, neutron 1.00867 u. Take $1 \text{ u} = 1.66 \times 10^{-27} \text{ kg}$, $c = 3.00 \times 10^8 \text{ m s}^{-1}$ and $1 \text{ MeV} = 1.60 \times 10^{-13} \text{ J}$.

(a) **State** what is meant by the **mass defect** of a nucleus. [1]

(b) **Determine** the mass difference of this reaction, in u. [2]

(c) **Determine** the energy released, in joules and in MeV. [3]

(d) **Determine** the energy released, in MeV, when 1.0 g of deuterium is fused this way. [3]

(e) **Explain** why fusion releases energy for light nuclei but fission releases energy for heavy ones, referring to **binding energy per nucleon**. [3]

Solutions

2.1 $E = mc^2$.

2.2 The difference between the **total mass of the separate nucleons** and the **mass of the nucleus**.

2.3 The energy needed to **separate a nucleus** into its individual nucleons (equivalent to the mass defect).

2.4 $14 + 4 = A + 1 \Rightarrow A = 17$.

3.1 B — the highest binding energy per nucleon.

3.2 $E = 0.030 \times 931 = 28 \text{ MeV}$.

3.3 Both light nuclei (fusing) and heavy nuclei (splitting) move **towards the peak** of the curve; this **increases** the binding energy per nucleon; the products are more tightly bound, so energy is **released**.

3.4 $E = mc^2 = (1.0 \times 10^{-29})(3.0 \times 10^8)^2 = 9.0 \times 10^{-13} \text{ J}$.

3.5 Mass lost $\Delta m = 209.98286 - (205.97444 + 4.00260) = 0.00582 \text{ u}$; energy released $Q = 0.00582 \times 931 = 5.4 \text{ MeV}$.

3.6 (a) The **difference** between the total mass of the separate nucleons and the mass of the assembled nucleus.

(b) Left: $2 \times 2.01410 = 4.02820 \text{ u}$; right: $3.01603 + 1.00867 = 4.02470 \text{ u}$; difference = 0.00350 u .

(c) $\Delta m = 0.00350 \times 1.66 \times 10^{-27} = 5.81 \times 10^{-30} \text{ kg}$; $E = \Delta mc^2 = 5.81 \times 10^{-30} \times (3.00 \times 10^8)^2 = 5.2 \times 10^{-13} \text{ J}$; dividing by 1.60×10^{-13} gives 3.3 MeV .

(d) 1.0 g of deuterium contains $\frac{1.0 \times 10^{-3}}{2.01410 \times 1.66 \times 10^{-27}} = 3.0 \times 10^{23}$ nuclei; the reaction consumes **two** nuclei per event, so there are 1.5×10^{23} events; total = $1.5 \times 10^{23} \times 3.3 = 4.9 \times 10^{23} \text{ MeV}$.

(e) Binding energy per nucleon rises steeply from hydrogen to a maximum near iron, then falls slowly for heavier nuclei. Energy is released whenever the products have a **greater** binding energy per nucleon than the reactants. So joining light nuclei moves up the curve and releases energy, while splitting a heavy nucleus moves up it from the other side and does the same.