

2.1 Equations of motion

Name: _____ Class: _____ Date: _____

Total: 49 marks**KEY WORDS** velocity 速度 • acceleration 加速度 • air resistance 空气阻力 • speed 速率 • distance 距离

Objective

Build the skills to answer exam questions on **uniformly accelerated motion** 匀加速运动 — choosing and using the SUVAT equations, reading motion graphs, and solving free-fall and projectile problems.

You must be able to:

- find velocity from a displacement–time gradient, and acceleration + displacement from a velocity–time graph
- choose and use the four SUVAT equations for straight-line motion
- solve free-fall problems ($g = 9.81 \text{ m s}^{-2}$, no air resistance)
- treat the horizontal and vertical motion of a projectile independently
- **describe** the experiment that measures the acceleration of free fall
- **sketch** a motion graph with both axes drawn and labelled

1 Worked examples

Study these first. Each one shows the **method** 方法 for a question type used later — examiners give marks for showing the working, not only the final number.

■ Choosing a SUVAT equation

The five symbols are u (start velocity), v (final velocity), a (acceleration), s (displacement) and t (time). Each equation uses **four** of them and leaves one out:

equation	leaves out
$v = u + at$	s
$s = \frac{1}{2}(u + v)t$	a
$s = ut + \frac{1}{2}at^2$	v
$v^2 = u^2 + 2as$	t

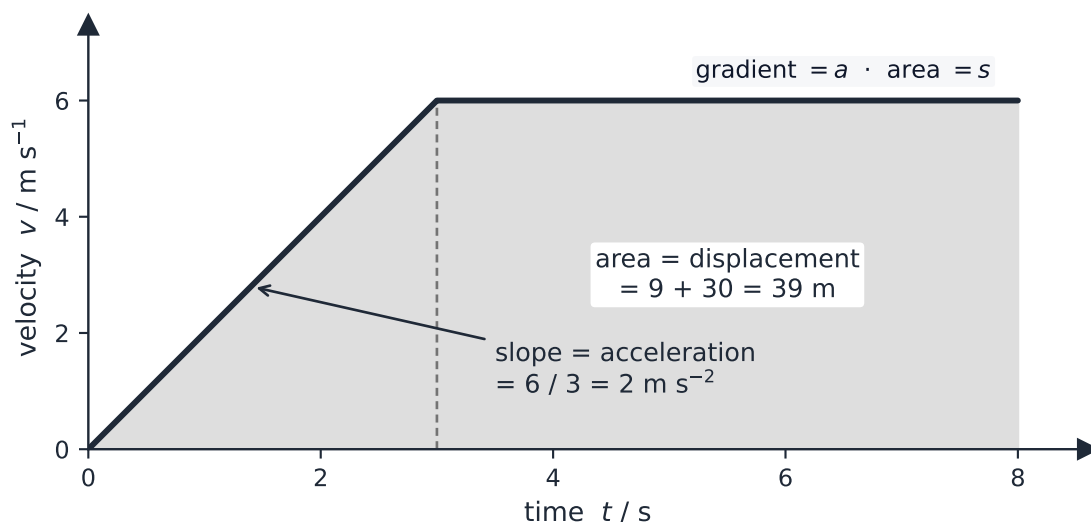
Method: list what you know and what you want, then pick the row that leaves out the symbol you don't have.

Example. A cyclist speeds up uniformly from 4.0 to 13 m s^{-1} in 6.0 s : so $u = 4.0$, $v = 13$, $t = 6.0$; find a , then s .

- For a (no s): $v = u + at \Rightarrow a = \frac{v - u}{t} = \frac{13 - 4.0}{6.0} = 1.5 \text{ m s}^{-2}$.
- For s (no a): $s = \frac{1}{2}(u + v)t = \frac{1}{2}(4.0 + 13)(6.0) = 51 \text{ m}$.

■ Reading a velocity–time graph

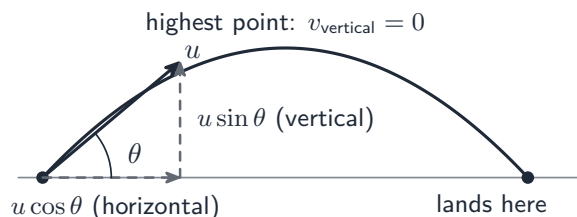
On a velocity–time graph the **gradient** 斜率 (steepness) is the acceleration, and the **area** under the line is the displacement.



(For an *acceleration*–time graph, the area under the line is instead the *change in velocity*.)

■ A projectile

Horizontal and vertical motion are **independent** — they share only the time t , so solve each as its own SUVAT problem. For a launch at angle θ , first **resolve** the speed: horizontal $u \cos \theta$ (stays constant) and vertical $u \sin \theta$ (changes).



Worked case — a ball thrown **horizontally** at 12 m s^{-1} from a cliff 25 m high ($g = 9.81 \text{ m s}^{-2}$):

- Vertical (start 0): $25 = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{2 \times 25/9.81} = 2.3 \text{ s}$.
- Horizontal: $x = 12 \times 2.3 = 27 \text{ m}$.

■ Measuring the acceleration of free fall

The syllabus asks you to **describe** this experiment, so learn it as five steps:

1. A steel ball is held by an **electromagnet** above a **trapdoor** switch, a measured height h above it.
2. Switching the magnet off starts an electronic **timer** and releases the ball at the same instant.
3. The ball opens the trapdoor on landing, which **stops** the timer, giving the fall time t .
4. The ball falls from rest, so $h = \frac{1}{2}gt^2$. Repeat for **several heights**, plot h against t^2 , and the **gradient** is $\frac{1}{2}g$.
5. Measure h from the **bottom** of the ball to the trapdoor, and repeat each timing — the graph, not a single pair of readings, is what removes the random error.

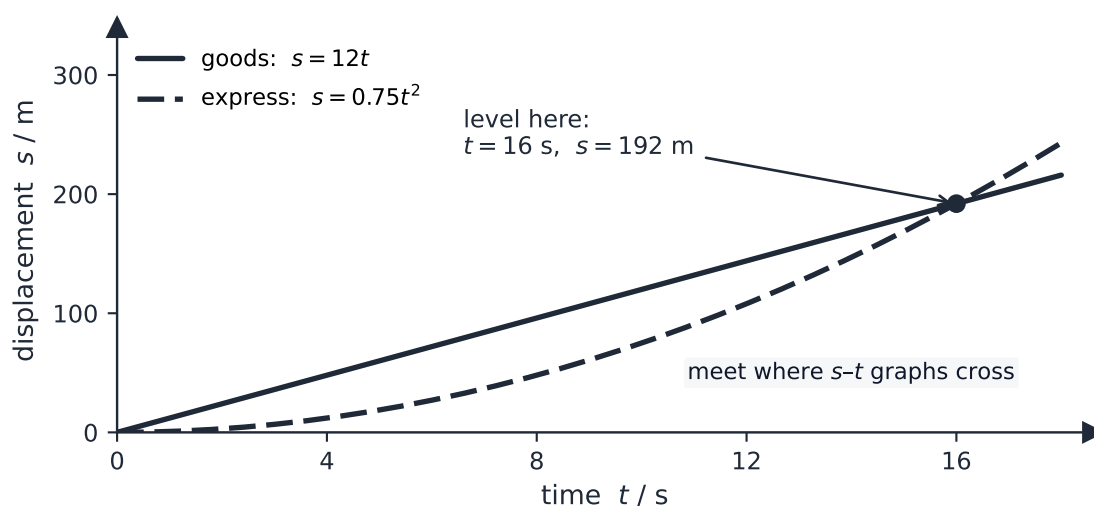
A steel ball is used because **air resistance** on it is negligible; the graph is a straight line **through the origin** only if there is no systematic timing delay.

■ Two objects meeting

Write a displacement equation for each object using the same start time and positive direction, then set the displacements equal.

A goods train moves at a constant 12 m s^{-1} . As it passes a signal, an express train starts from rest there with acceleration 1.5 m s^{-2} .

- Goods: $s = 12t$. Express: $s = \frac{1}{2}(1.5)t^2 = 0.75t^2$.
- Level when $12t = 0.75t^2 \Rightarrow t = \frac{12}{0.75} = 16 \text{ s}$ ($s = 12 \times 16 = 192 \text{ m}$).



2 Practice

Now apply the methods above.

2.1 For each quantity, write **scalar** 标量 or **vector** 矢量, and state its SI unit: (a) velocity (b) acceleration. [2]

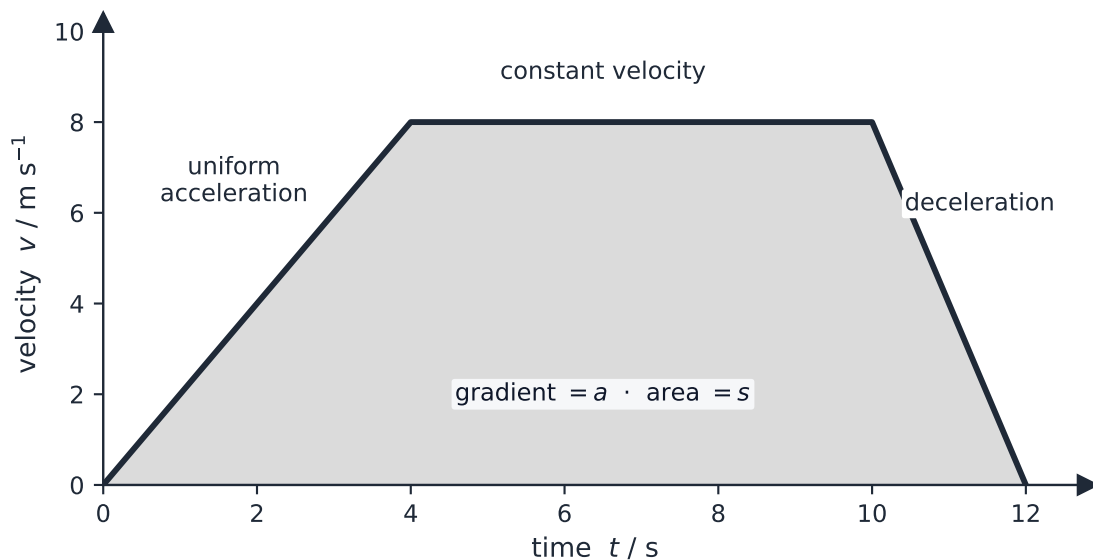
quantity	scalar or vector?	SI unit
velocity		
acceleration		

2.2 A car travelling at 24 m s^{-1} brakes uniformly at 3.0 m s^{-2} until it stops. Find the braking distance. (*Hint: which SUVAT equation links u , v , a and s ?*) [3]

2.3 The displacement–time graph of a cyclist is a straight line passing through (2.0 s, 12 m) and (8.0 s, 42 m). **Determine** her velocity, and state how you know her acceleration is zero. [3]

2.4 Describe an experiment to determine the acceleration of free fall using a falling object. State the measurements you would take, how you would use them, and one way to reduce the uncertainty. [4]

2.5 The velocity–time graph shows a tram’s journey.



Velocity–time graph for the tram’s three-phase journey

- (a) Find the acceleration during the first 4 s. [1]

- (b) Find the total displacement of the tram. (*Hint: split the area into a triangle, a rectangle and a triangle.*) [3]

3 Exam-style questions

3.1 A stone is thrown vertically upwards at 15 m s^{-1} . Take $g = 9.81 \text{ m s}^{-2}$ and ignore air resistance. Its height above the launch point after 2.0 s is closest to: [1]

- A 10 m
- B 20 m
- C 30 m
- D 50 m

3.2 The area under an **acceleration–time** graph gives which quantity? [1]

- A displacement

- **B** change in velocity
 - **C** distance
 - **D** force
-

3.3 A train starts from rest and accelerates uniformly at 0.50 m s^{-2} for 40 s.

(a) Calculate the speed it reaches. [2]

(b) Calculate the distance travelled during these 40 s. [2]

(c) State one assumption you have made in your calculation. [1]

3.4 A ball is kicked from level ground at 20 m s^{-1} , at 30° above the horizontal. Take $g = 9.81 \text{ m s}^{-2}$ and ignore air resistance.

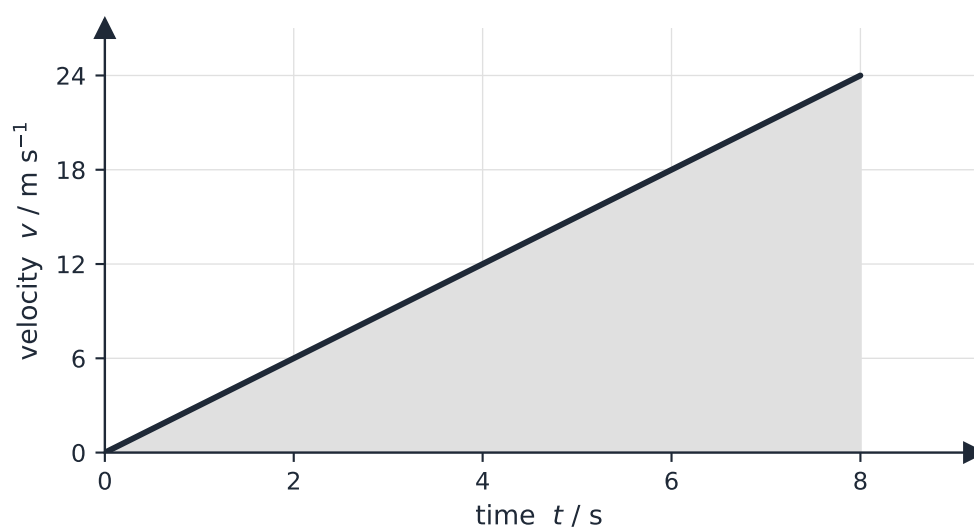
(a) Show that the vertical **component** 分量 of the initial velocity is about 10 m s^{-1} . [1]

(b) Calculate the time the ball is in the air before it lands. [2]

(c) Calculate the horizontal distance it travels (its range).

[2]

3.6 A drone of mass 1.8 kg takes off from the ground and rises vertically. The graph shows its velocity for the first 8.0 s of the flight.



(a) **Define** acceleration.

[1]

(b) **Determine** the acceleration of the drone.

[2]

(c) **Show that** the drone rises 96 m in these 8.0 s.

[2]

(d) Calculate the gain in **gravitational potential energy** of the drone. [2]

(e) Calculate the gain in **kinetic energy** of the drone. [2]

(f) **Determine** the average useful power output of the drone's motors during these 8.0 s. [2]

(g) In the space below, **sketch** the variation with time of the **displacement** of the drone over the same 8.0 s. **Draw** and **label** both axes. [3]

(h) A second drone of mass 2.7 kg takes off and follows exactly the same velocity–time graph. **State and explain** whether the average useful power output of its motors is less than, the same as, or greater than your answer to (f). [2]

3.5 A car passes a stationary motorcyclist at a constant 18 m s^{-1} . At that instant the motorcyclist sets off from rest with a uniform acceleration of 3.0 m s^{-2} .

(a) Calculate the time the motorcyclist takes to catch up with the car. [3]

(b) Calculate how far each has travelled at that moment. [1]

(c) Calculate the motorcyclist's speed when she catches the car. [1]

Solutions

2.1 velocity — vector, m s^{-1} ; acceleration — vector, m s^{-2} .

2.2 Use $v^2 = u^2 + 2as$ with $v = 0$: $0 = 24^2 + 2(-3.0)s \Rightarrow s = 576/6.0 = 96 \text{ m}$.

2.3 velocity = gradient = $\frac{42 - 12}{8.0 - 2.0} = \frac{30}{6.0} = 5.0 \text{ m s}^{-1}$; the graph is a **straight** line, so the gradient — and therefore the velocity — never changes, and acceleration is the rate of change of velocity, so it is zero.

2.4 Release a steel ball from an electromagnet a measured height h above a trapdoor switch; switching the magnet off starts an electronic timer and releases the ball, and the ball opens the trapdoor to stop it, giving t ; use $h = \frac{1}{2}gt^2$, or better plot h against t^2 and take $g = 2 \times \text{gradient}$; reduce uncertainty by repeating each timing and by using several heights so a graph averages the readings.

2.5 (a) gradient = $8/4 = 2.0 \text{ m s}^{-2}$.

(b) triangle $\frac{1}{2}(4)(8) = 16$; rectangle $(6)(8) = 48$; triangle $\frac{1}{2}(2)(8) = 8$; total = $16 + 48 + 8 = 72 \text{ m}$.

3.1 A. $s = (15)(2.0) - \frac{1}{2}(9.81)(2.0)^2 = 30 - 19.6 = 10.4 \text{ m}$, closest to 10 m .

3.2 B. The area under an acceleration–time graph is the change in velocity.

3.3 (a) $v = u + at = 0 + (0.50)(40) = 20 \text{ m s}^{-1}$.

(b) $s = \frac{1}{2}at^2 = \frac{1}{2}(0.50)(40)^2 = 400 \text{ m}$. [or $s = \frac{1}{2}(0 + 20)(40)$]

(c) e.g. the acceleration stays constant / the track is straight.

3.4 (a) $u_V = 20 \sin 30^\circ = (20)(0.50) = 10 \text{ m s}^{-1}$.

(b) time to the top = $u_V/g = 10/9.81 = 1.02 \text{ s}$; total time = $2 \times 1.02 = 2.0 \text{ s}$.

(c) $u_H = 20 \cos 30^\circ = 17.3 \text{ m s}^{-1}$; range = $u_H \times t = 17.3 \times 2.04 = 35 \text{ m}$.

3.6 (a) Acceleration is the **rate of change of velocity**.

(b) gradient = $\frac{24 - 0}{8.0 - 0} = 3.0 \text{ m s}^{-2}$.

(c) height = area under the line = $\frac{1}{2}(8.0)(24) = 96 \text{ m}$. [or $s = \frac{1}{2}at^2 = \frac{1}{2}(3.0)(8.0)^2$]

(d) $\Delta E_P = mgh = 1.8 \times 9.81 \times 96 = 1.7 \times 10^3 \text{ J}$.

(e) $\Delta E_K = \frac{1}{2}mv^2 = \frac{1}{2}(1.8)(24)^2 = 5.2 \times 10^2 \text{ J}$.

(f) $P = \frac{\text{total energy gained}}{t} = \frac{1.7 \times 10^3 + 5.2 \times 10^2}{8.0} = 2.8 \times 10^2 \text{ W}$.

(g) Axes drawn and labelled *displacement / m* (vertical) and *time / s* (horizontal); a curve starting at the origin; getting **steeper** with time (a parabola, since $s \propto t^2$), reaching 96 m at 8.0 s .

(h) **Greater.** The graph is the same, so it gains the same height and the same speed, but both $\Delta E_P = mgh$ and $\Delta E_K = \frac{1}{2}mv^2$ are **proportional to the mass**, so a heavier drone gains more energy in the same 8.0 s .

3.5 (a) car: $s = 18t$; motorcyclist: $s = \frac{1}{2}(3.0)t^2 = 1.5t^2$. They are level when $18t = 1.5t^2 \Rightarrow t = 18/1.5 = 12 \text{ s}$.

(b) $s = (18)(12) = 216 \text{ m}$.

(c) $v = at = (3.0)(12) = 36 \text{ m s}^{-1}$.