19.3 Discharging a capacitor

Name: ______________ Class: __________ Date: ____________ **Total: 29 marks**

KEY WORDS time constant 时间常数 • exponential decay 指数衰减 • resistor 电阻器 • resistance 电阻 • capacitor 电容器

Objective

Build the skills to answer exam questions on a **capacitor discharging** through a resistor.

You must be able to:

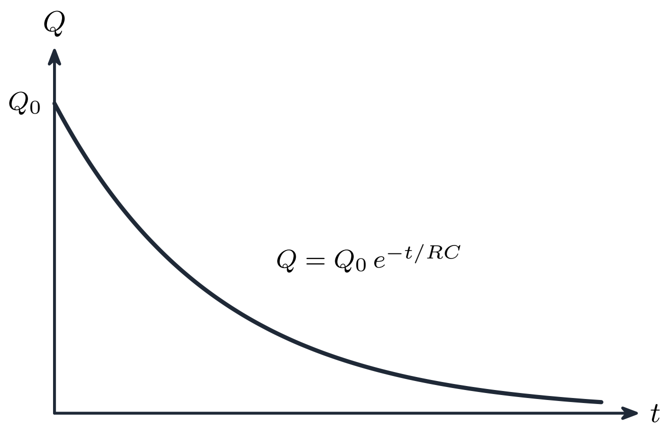
- use the **time constant** $\tau = RC$
- use $x = x_0 e^{-t/RC}$ for charge, p.d. or current
- interpret the discharge graphs

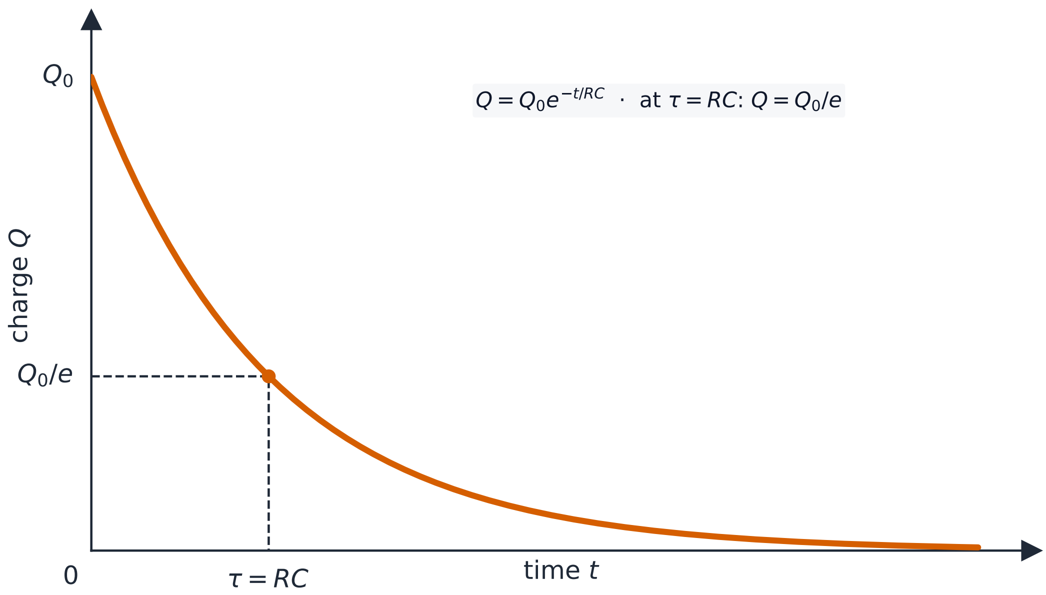
1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Exponential decay

When a charged capacitor discharges through a resistor, the charge (and V, I) decays **exponentially**:





Exponential discharge of a capacitor

$$Q = Q_0 e^{-t/RC}, \quad V = V_0 e^{-t/RC}, \quad I = I_0 e^{-t/RC}.$$

The **time constant** $\tau = RC$ is the time for the charge to fall to $\frac{1}{e} \approx 37\%$ of its initial value.

Worked example. 470 μF through 2.0 kΩ: $\tau = RC = (2000)(470 \times 10^{-6}) = 0.94$ s.

■ Finding the time — take logs

To find the **time** for the charge (or V, or I) to fall to a given value, take the **natural log** of both sides of $V = V_0 e^{-t/RC}$:

$$\ln \frac{V}{V_0} = -\frac{t}{RC} \quad \Longrightarrow \quad t = -RC \ln \frac{V}{V_0}.$$

Worked example. For the capacitor above (charged to 6.0 V, $\tau = 0.94$ s), the time for the p.d. to fall to 2.0 V is $t = -0.94 \ln \dfrac{2.0}{6.0} = -0.94 \times (-1.10) = 1.0$ s.

2 Practice

Now apply the methods above.

2.1 Write the formula for the **time constant**. [1]

2.2 A 100 μF capacitor discharges through a 10 kΩ resistor. Find the **time constant**. [2]

2.3 State what happens to the **charge** as a capacitor discharges. [1]

2.4 Write the discharge equation for **charge**. [1]

3 Exam-style questions

3.1 The time constant RC is the time for the charge to fall to: [1]

- **A** zero
- **B** half its initial value
- **C** $1/e$ (about 37%) of its initial value
- **D** $1/2e$ of its initial value

3.2 A 470 μF capacitor charged to 6.0 V discharges through a 2.0 kΩ resistor. Calculate (a) the time constant, (b) the p.d. after 1.0 s. [4]

3.3 Sketch a graph of **charge against time** for a discharging capacitor. [2]

3.4 State what happens to the time constant if the **resistance** is doubled. [1]

3.5 A 220 μF capacitor charged to 9.0 V discharges through a 15 kΩ resistor. Calculate the **time** for the p.d. to fall to 3.0 V. [4]

3.6 A capacitor C is charged and then discharged through a resistor R. A data logger records the p.d. V_R across the resistor and the p.d. V_C across the capacitor. At $t = 0$ the p.d. across the capacitor is 9.0 V, and the initial current is 1.8 mA. The p.d. across the capacitor falls to 3.3 V after 12 s.

(a) **State** the relationship between V_C and V_R during the discharge, and give a reason. [2]

(b) **Determine** the resistance R. [2]

(c) **Determine** the time constant τ of the circuit. [3]

(d) Hence **determine** the capacitance C, in μF. [2]

(e) **Explain**, in terms of the current in the circuit, why the decay of the charge on the capacitor is **exponential**. [3]

Solutions

2.1 $\tau = RC$.

2.2 $\tau = RC = (10 \times 10^3)(100 \times 10^{-6}) = 1.0$ s.

2.3 It **decreases exponentially** towards zero.

2.4 $Q = Q_0 e^{-t/RC}$.

3.1 **C** — $1/e$ (about 37%).

3.2 (a) $\tau = RC = (2.0 \times 10^3)(470 \times 10^{-6}) = 0.94$ s. (b) $V = V_0 e^{-t/RC} = 6.0\, e^{-1.0/0.94} = 6.0 \times 0.345 = 2.1$ V.

3.3 A curve starting at the initial charge and **decaying exponentially** towards zero, with the same shape (never quite reaching zero).

3.4 It **doubles** ($\tau = RC$).

3.5 $\tau = RC = (15 \times 10^3)(220 \times 10^{-6}) = 3.3$ s; $t = -RC \ln \dfrac{V}{V_0} = -3.3 \ln \dfrac{3.0}{9.0} = -3.3 \times (-1.10) = 3.6$ s.

3.6 (a) They are **equal** at every instant, $V_C = V_R$, because the capacitor and resistor form a single loop with no other source, so the p.d. driving the current is the capacitor's own p.d..
(b) $R = \dfrac{V}{I} = \dfrac{9.0}{1.8 \times 10^{-3}} = 5.0 \times 10^3\ \Omega$.
(c) $V = V_0 e^{-t/\tau}$, so $\dfrac{3.3}{9.0} = e^{-12/\tau}$; $\ln(0.367) = -\dfrac{12}{\tau}$, giving $-1.003 = -\dfrac{12}{\tau}$ and $\tau = 12$ s. (The p.d. has fallen to $1/e$ of its start, which is what the time constant means.)
(d) $\tau = RC$, so $C = \dfrac{12}{5.0 \times 10^3} = 2.4 \times 10^{-3}$ F $= 2.4 \times 10^3\ \mu$F.
(e) The current is $I = \dfrac{V_C}{R}$, and $V_C \propto Q$, so the current — the rate at which charge leaves — is **proportional to the charge still on the capacitor**. A quantity whose rate of decrease is proportional to itself decays exponentially; so as the charge falls the current falls with it, the discharge slows, and Q never quite reaches zero.