19.2 Energy stored in a capacitor

Name: ______________ Class: __________ Date: ____________ **Total: 23 marks**

KEY WORDS energy 能量 • capacitor 电容器

Objective

Build the skills to answer exam questions on the **energy stored in a capacitor**.

You must be able to:

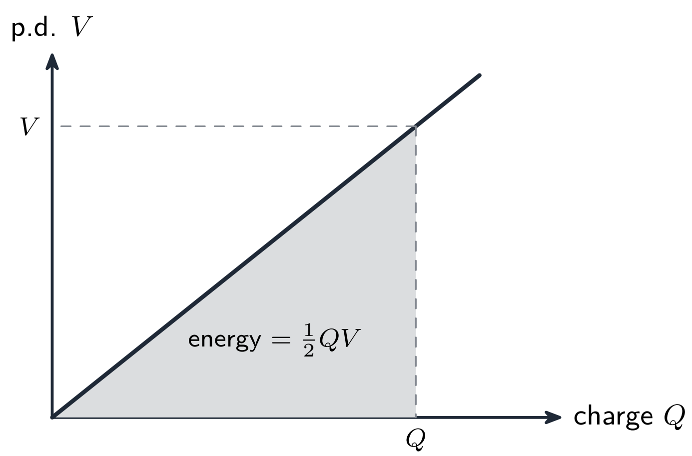
- find energy from the **area under the potential–charge graph**
- use $W = \frac{1}{2}QV = \frac{1}{2}CV^2 = \dfrac{Q^2}{2C}$

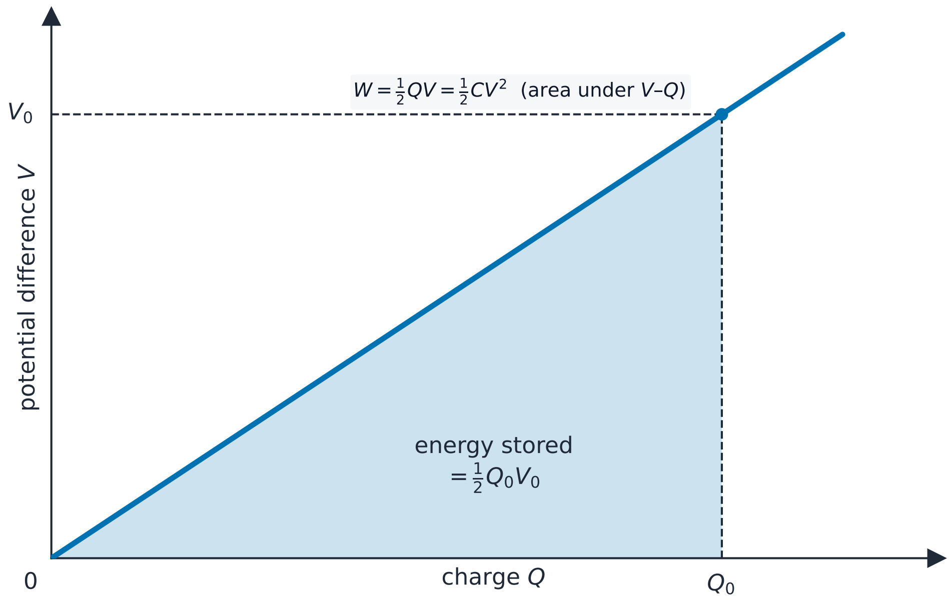
1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

Energy stored

The energy is the **area under the V–Q graph** — a triangle:





Energy stored is the area under the Q-V graph

$$W = \tfrac{1}{2}QV = \tfrac{1}{2}CV^2 = \frac{Q^2}{2C}.$$

The factor $\frac{1}{2}$ appears because the **average** p.d. during charging is $V/2$ (it grows from 0 to V).

Worked example. 100 μF at 12 V: $W = \frac{1}{2}CV^2 = \frac{1}{2}(100\times10^{-6})(12)^2 = 7.2\times10^{-3}$ J.

2 Practice

Now apply the methods above.

2.1 Write the **three** forms of the capacitor energy formula. [1]

2.2 Find the energy stored in a 100 μF capacitor charged to 12 V. [2]

2.3 State what the **area** under a potential–charge graph represents. [1]

3 Exam-style questions

3.1 The energy stored in a capacitor is: [1]

- **A** QV
- **B** $\frac{1}{2}QV$
- **C** CV
- **D** Q^2C

3.2 A 220 μF capacitor stores 4.0 mJ of energy. Calculate the **voltage** across it. [3]

3.3 Explain why the energy stored is $\frac{1}{2}QV$ and **not** QV. [2]

3.5 A defibrillator stores energy in a 32 μF capacitor charged to 2.5 kV, and delivers it to a patient in a pulse lasting 8.0 ms.

(a) **Determine** the energy stored by the capacitor when fully charged. [2]

(b) **Determine** the charge stored. [2]

(c) In the pulse, 75% of the stored energy is delivered to the patient. **Determine** the average power delivered. [3]

(d) The capacitor is charged instead to 1.8 kV. **Determine** the percentage by which the stored energy falls, and **explain** why it is not the same percentage as the fall in p.d. [3]

(e) On the same axes, **sketch** how the energy stored varies with the p.d. across the capacitor, and how the **charge** stored varies with it. **Label** each line. [3]

Solutions

2.1 $W = \frac{1}{2}QV = \frac{1}{2}CV^2 = \dfrac{Q^2}{2C}$.

2.2 $W = \frac{1}{2}CV^2 = \frac{1}{2}(100 \times 10^{-6})(12)^2 = 7.2 \times 10^{-3}$ J.

2.3 The **energy stored** in the capacitor.

3.1 **B** — $\frac{1}{2}QV$.

3.2 $W = \frac{1}{2}CV^2 \Rightarrow V = \sqrt{\dfrac{2W}{C}} = \sqrt{\dfrac{2(4.0 \times 10^{-3})}{220 \times 10^{-6}}} = 6.0$ V.

3.3 As the capacitor charges, the p.d. rises from 0 to V; the **average** p.d. is $V/2$, so the work done is $Q \times (V/2) = \frac{1}{2}QV$, not QV.

3.5 (a) $E = \frac{1}{2}CV^2 = \frac{1}{2}(32 \times 10^{-6})(2500)^2 = 100$ J.
(b) $Q = CV = 32 \times 10^{-6} \times 2500 = 8.0 \times 10^{-2}$ C.
(c) Energy delivered $= 0.75 \times 100 = 75$ J; $P = \dfrac{E}{t} = \dfrac{75}{8.0 \times 10^{-3}} = 9.4 \times 10^3$ W. (About nine kilowatts, for eight milliseconds.)
(d) New energy $= \frac{1}{2}(32 \times 10^{-6})(1800)^2 = 51.8$ J, a fall of $\dfrac{100 - 51.8}{100} = 48\%$ — while the p.d. fell by only 28%, because $E \propto V^2$, so the fractional change in energy is roughly **twice** the fractional change in p.d..
(e) Charge: a **straight line through the origin**, since $Q = CV$. Energy: a **curve through the origin** getting steeper, since $E \propto V^2$. Both labelled, with axes labelled.