17.3 Damped and forced oscillations, resonance

Name: ____________ Class: ________ Date: __________ **Total: 23 marks**

KEY WORDS resonance 共振 • driving frequency 驱动频率 • natural frequency 固有频率 • forced oscillation 受迫振
• critical damping 临界阻尼

Objective

Build the skills to answer exam questions on **damping** and **resonance**.

You must be able to:

- describe **light**, **critical** and **heavy** damping and sketch their graphs
- explain **resonance** and the condition for it

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

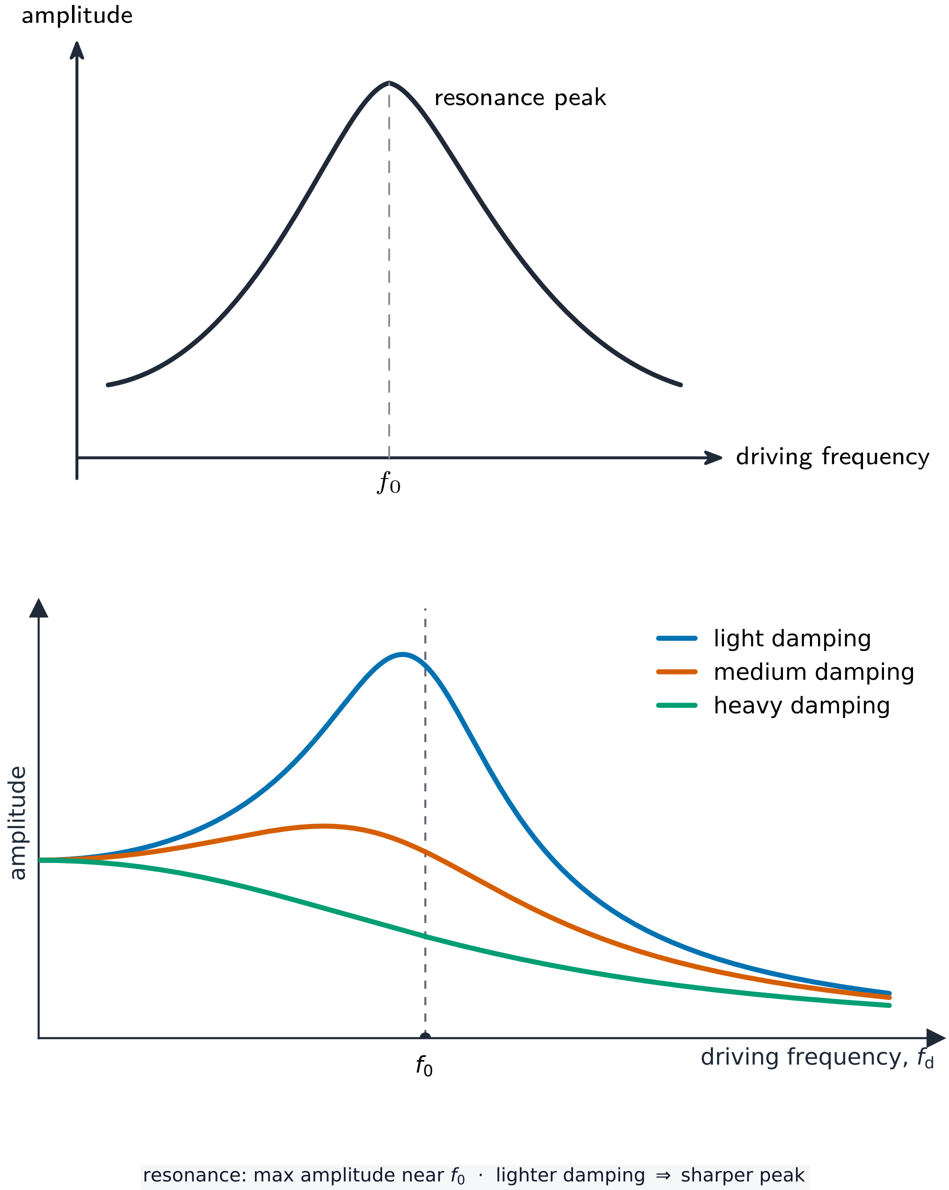
■ **Damping**

A **resistive force** removes energy, so the amplitude **shrinks** — this is **damping**:

- **light:** amplitude dies away slowly over many cycles;
- **critical:** returns to equilibrium **fastest** with **no overshoot**;
- **heavy:** returns slowly, no oscillation.

■ **Resonance**

A system **forced** to oscillate has the largest amplitude when the **driving frequency equals its natural frequency** f_0 — this is **resonance**:



The resonance curve and damping

More damping makes the resonance peak **lower and broader**.

2 Practice

Now apply the methods above.

2.1 State what causes **damping**. [1]

2.2 Name the **three** types of damping. [2]

2.3 State what is meant by **resonance**. [1]

2.4 State the condition for resonance to occur. [1]

3 Exam-style questions

3.1 **Critical** damping is the damping that: [1]

- **A** lets the system oscillate forever
- **B** returns the system to equilibrium fastest, without overshooting
- **C** returns the system to equilibrium most slowly
- **D** increases the amplitude

3.2 Describe how the **amplitude** of a forced oscillation changes as the driving frequency is increased through the natural frequency. [3]

3.3 Give **one** example each of where **light** damping and **critical** damping are useful. [2]

3.4 Explain what happens to the **resonance peak** when more damping is added. [2]

3.5 A machine part is mounted on rubber blocks. When the machine runs, the part is driven into oscillation, and a technician measures the amplitude of the part at each driving frequency.

(a) **State** the conditions needed for **resonance** to occur. [2]

(b) The amplitude peaks sharply at a driving frequency of 24 Hz. **State** what this tells you about the part, and **determine** its natural period. [2]

(c) The rubber blocks are replaced with stiffer ones, which **increase** the natural frequency of the part but also **increase** the damping. On one set of axes, **sketch** two curves of amplitude against driving frequency: the original mounting, labelled A, and the stiffer one, labelled B. [4]

(d) **Explain** why an engineer would prefer the stiffer, more heavily damped mounting even though the machine's own running frequency cannot be changed. [2]

Solutions

2.1 A **resistive force** (friction / drag / air resistance) removing energy from the oscillator.

2.2 Light, **critical** and **heavy** damping.

2.3 The condition where a forced oscillation reaches its **maximum amplitude**.

2.4 The **driving frequency equals the natural frequency** of the system.

3.1 B — returns to equilibrium fastest without overshooting.

3.2 The amplitude **rises** as the driving frequency approaches the natural frequency, reaching a **maximum at** f_0 (resonance), then **falls** again as the frequency increases past f_0.

3.3 Light damping — e.g. a pendulum/clock or musical instrument (sustained oscillation); critical damping — e.g. a galvanometer/voltmeter needle or car suspension/door closer (quick settle).

3.4 The peak becomes **lower** (smaller maximum amplitude) and **broader** (occurs over a wider range of frequencies).

3.5 (a) The system is driven by a **periodic force** whose frequency equals the **natural frequency** of the system, and there is a **maximum transfer of energy** to it, so its amplitude becomes maximum.
(b) The natural frequency of the part is 24 Hz; $T = \dfrac{1}{f} = \dfrac{1}{24} = 0.042$ s.
(c) Curve A: a sharp peak at 24 Hz. Curve B: its peak is at a **higher** frequency, is **lower** in amplitude, and is **broader** — heavier damping flattens and widens the resonance curve; both curves start at a non-zero amplitude at zero frequency and fall away at high frequency.
(d) The machine's running frequency is now **further from** the part's natural frequency, so the part is driven well away from resonance; and the extra damping lowers the amplitude at every frequency, so even at resonance the oscillation would be smaller and less likely to cause damage.