A-LEVEL PHYSICS • EXERCISE SHEET

17.2 Energy in simple harmonic motion

Name: ______________ Class: __________ Date: ____________ **Total: 29 marks**

KEY WORDS kinetic energy 动能 • damping 阻尼 • displacement 位移 • equilibrium 平衡 • amplitude 振幅

Objective

Build the skills to answer exam questions on **energy in simple harmonic motion**.

You must be able to:

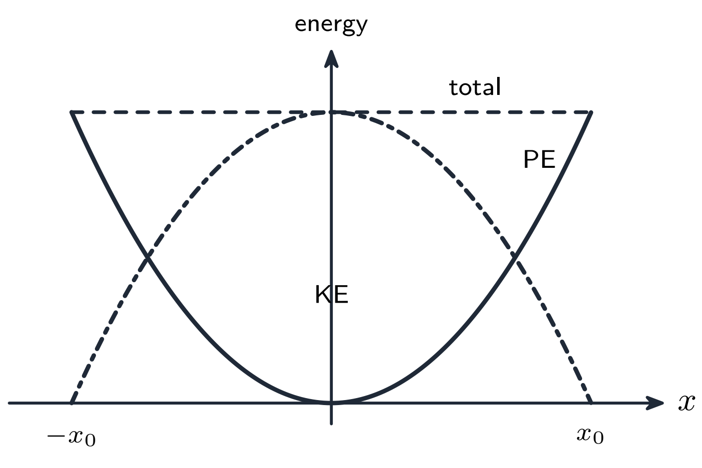
- describe the **KE** ↔ **PE** interchange during SHM
- use $E = \frac{1}{2}m\omega^2 x_0^2$ for the total energy

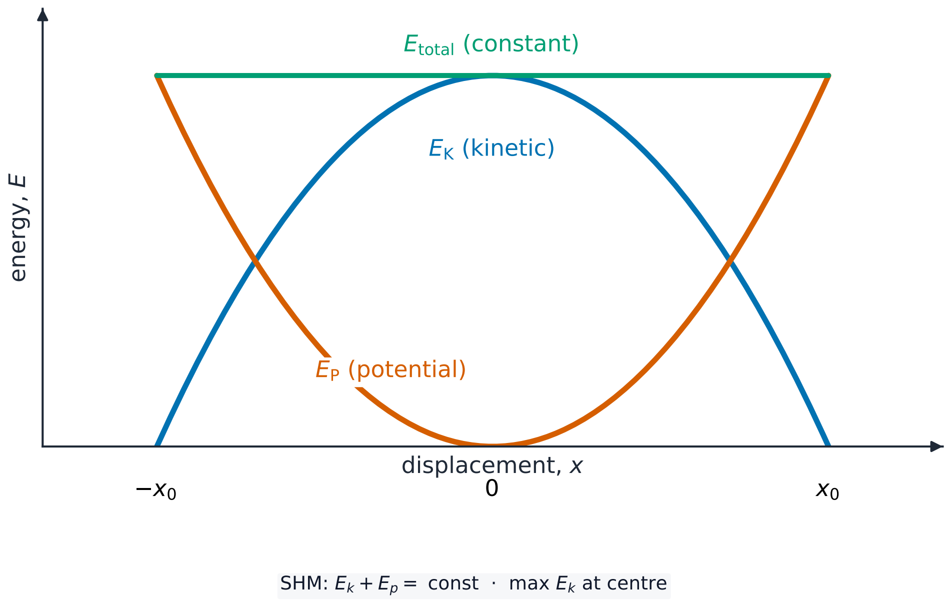
1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Energy interchange

KE and PE swap as the oscillator moves; with no damping the **total energy is constant**:





Energy interchange in simple harmonic motion

- at $x = 0$: KE is **maximum**, PE minimum;
- at $x = \pm x_0$: KE $= 0$, PE **maximum**.

Total energy

$$E = \tfrac{1}{2}mv_{\text{max}}^2 = \tfrac{1}{2}m\omega^2 x_0^2.$$

So $E \propto x_0^2$ (doubling the amplitude gives **four times** the energy) and $E \propto \omega^2$.

Energy varies at twice the frequency

In one full oscillation the object passes through the centre (KE maximum) **twice**, so the **KE and PE vary at twice** the frequency of the displacement. On a graph of KE against **time**, one repeat of the KE pattern takes **half** the oscillation period — so if the KE graph repeats every t_E, the oscillation period is $T = 2t_E$.

2 Practice

Now apply the methods above.

2.1 Describe the **energy interchange** during SHM. [2]

2.2 State where the **kinetic energy** is maximum and where the **potential energy** is maximum. [2]

2.3 Write the formula for the **total energy** of a simple harmonic oscillator. [1]

3 Exam-style questions

3.1 In SHM, the total energy is proportional to: [1]

- **A** the amplitude x_0
- **B** the amplitude squared x_0^2
- **C** $1/x_0$
- **D** the period T

3.2 A 0.20 kg mass oscillates with $\omega = 10$ rad s^{-1} and amplitude 0.050 m. Calculate the **total energy**. [3]

3.3 Sketch how the **kinetic energy** and **potential energy** vary with displacement in

SHM. [2]

3.4 State what happens to the total energy if the **amplitude is doubled**. [1]

3.5 A graph of the **kinetic energy** of an oscillator against **time** repeats every 0.40 s.

(a) State the **period** of the oscillation, explaining your answer. [2]

(b) Calculate the **angular frequency** ω of the oscillation. [2]

3.6 A mass hangs on a spring and oscillates vertically with simple harmonic motion. A data logger records its **kinetic energy** against time, giving a curve that repeats every 0.40 s. The mass is 0.25 kg and the amplitude of the oscillation is 4.0 cm.

(a) **Determine** the period of the oscillation of the **mass**, and explain why it is not 0.40 s. [2]

(b) **Determine** the angular frequency of the oscillation. [2]

(c) **Determine** the total energy of the oscillation. [3]

(d) **Determine** the speed of the mass when its displacement is 2.0 cm. [3]

(e) The oscillation is **lightly damped**. **State and explain** what happens to the time between successive maxima of the kinetic-energy curve, and to the height of those maxima. [3]

Solutions

2.1 Energy is continually transferred between **kinetic** and **potential** forms; with no damping the **total** stays constant.

2.2 KE is maximum at the **equilibrium** ($x = 0$); PE is maximum at the **extremes** ($x = \pm x_0$).

2.3 $E = \frac{1}{2}m\omega^2 x_0^2$.

3.1 **B** — the amplitude squared.

3.2 $E = \frac{1}{2}m\omega^2 x_0^2 = \frac{1}{2}(0.20)(10)^2(0.050)^2 = 0.025$ J.

3.3 KE: a **downward** parabola, maximum at $x = 0$, zero at $\pm x_0$; PE: an **upward** parabola, zero at $x = 0$, maximum at $\pm x_0$; their sum is constant.

3.4 It becomes **four times** larger ($E \propto x_0^2$).

3.5 (a) The KE varies at **twice** the oscillation frequency (the object passes the centre twice per cycle), so the oscillation period is **twice** the KE-graph period: $T = 2 \times 0.40 = 0.80$ s.

3.5 (b) $\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{0.80} = 7.9$ rad s^{-1}.

3.6 (a) The kinetic energy is zero at **both** ends of the swing and maximum at the centre twice per cycle, so the energy curve repeats **twice** per oscillation; the period of the mass is therefore $2 \times 0.40 = 0.80$ s.
(b) $\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{0.80} = 7.9$ rad s^{-1}.
(c) The total energy equals the maximum kinetic energy, $E = \frac{1}{2}m\omega^2 x_0^2 = \frac{1}{2}(0.25)(7.85)^2(0.040)^2 = 1.2 \times 10^{-2}$ J.
(d) $v = \omega\sqrt{x_0^2 - x^2} = 7.85\sqrt{(0.040)^2 - (0.020)^2} = 7.85 \times 0.0346 = 0.27$ m s^{-1}.
(e) Light damping barely changes the period, so the time between maxima stays about 0.40 s. The maxima get **lower** with time, because the amplitude decays and the total energy is proportional to the **square** of the amplitude, so the energy falls faster than the amplitude does.