A-LEVEL PHYSICS • EXERCISE SHEET

17.1 Simple harmonic oscillations

Name: ______________ Class: __________ Date: ____________ **Total: 34 marks**

KEY WORDS simple harmonic motion 简谐运动 • acceleration 加速度 • displacement 位移 • period 周期 • frequency 频率

Objective

Build the skills to answer exam questions on **simple harmonic motion** — the defining equation, the solutions, and the graphs.

You must be able to:

- use the SHM condition $a = -\omega^2 x$ and $x = x_0 \sin \omega t$
- use $v = \pm\omega\sqrt{x_0^2 - x^2}$ and $v_{\max} = \omega x_0$
- interpret the displacement, velocity and acceleration graphs

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

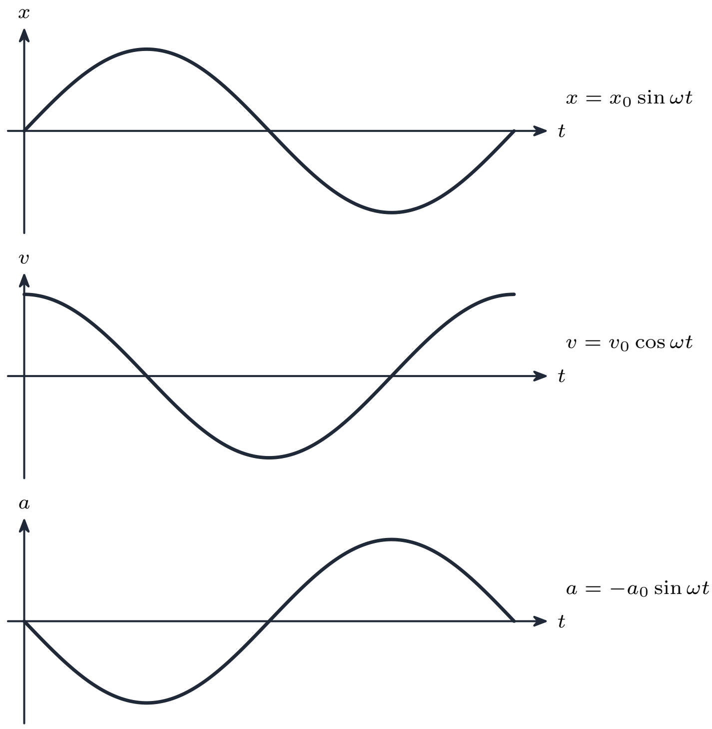
■ What SHM is

A particle moves with **SHM** when its acceleration is **proportional to displacement** and **directed back** to equilibrium:

$$a = -\omega^2 x, \qquad x = x_0 \sin \omega t, \qquad T = \frac{2\pi}{\omega}.$$

■ Velocity and the graphs

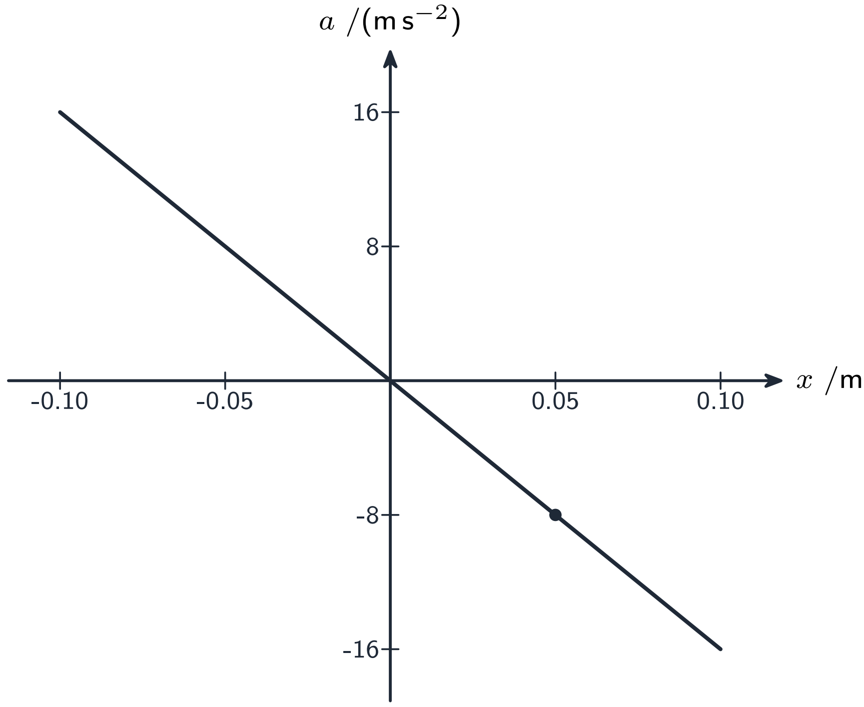
$$v = v_0 \cos \omega t, \qquad v = \pm\omega\sqrt{x_0^2 - x^2}, \qquad v_{\max} = \omega x_0, \qquad a_{\max} = \omega^2 x_0.$$



Displacement and acceleration are **antiphase**; velocity leads displacement by a **quarter cycle**.

■ The acceleration–displacement graph

Because $a = -\omega^2 x$, a graph of a against x is a **straight line through the origin** with a **negative gradient** of magnitude ω^2:



This is the **test for SHM**: $a \propto x$ (straight line through the origin) and **opposite in direction** (negative gradient). Read the gradient to find ω: gradient $= -\omega^2$, so $\omega = \sqrt{|\text{gradient}|}$.

Worked example. The line above passes through (0.05 m, -8 m s^{-2}), so gradient $= -8/0.05 = -160\,\text{s}^{-2} = -\omega^2$; thus $\omega = \sqrt{160} = 12.6$ rad s^{-1} and $T = 2\pi/\omega = 0.50$ s.

2 Practice

Now apply the methods above.

2.1 State the **two** conditions for simple harmonic motion. [2]

2.2 Define the **amplitude** of an oscillation. [1]

2.3 A particle has $\omega = 5.0$ rad s^{-1} and amplitude 0.040 m. Find its **maximum speed.** [2]

2.4 State the **phase relationship** between displacement and acceleration in SHM. [1]

3 Exam-style questions

3.1 In simple harmonic motion, the acceleration is: [1]

- **A** proportional to displacement and in the same direction
- **B** proportional to displacement and in the opposite direction
- **C** constant
- **D** always zero

3.2 A mass on a spring oscillates with period 0.50 s and amplitude 0.030 m. Find (a) the angular frequency, (b) the maximum speed, (c) the maximum acceleration. [4]

3.3 A particle in SHM has amplitude 0.10 m and $\omega = 8.0$ rad s^{-1}. Find its **speed** when

$x = 0.060$ m. [3]

3.4 Sketch the **displacement–time** and **acceleration–time** graphs for SHM, and state their phase relationship. [3]

3.5 An object oscillates. A graph of its acceleration a against displacement x is a straight line through the origin passing through (0.05 m, −8.0 m s^{-2}).

(a) Explain how this graph shows the motion is **simple harmonic**. [2]

(b) Use the **gradient** to find the angular frequency ω and the period T. [3]

3.6 A small ball hangs on a spring and is set oscillating vertically. A motion sensor records its displacement, and the graph obtained is a smooth curve whose successive maximum displacements are equal, with a maximum displacement of x_0 and 8 complete oscillations in 6.4 s.

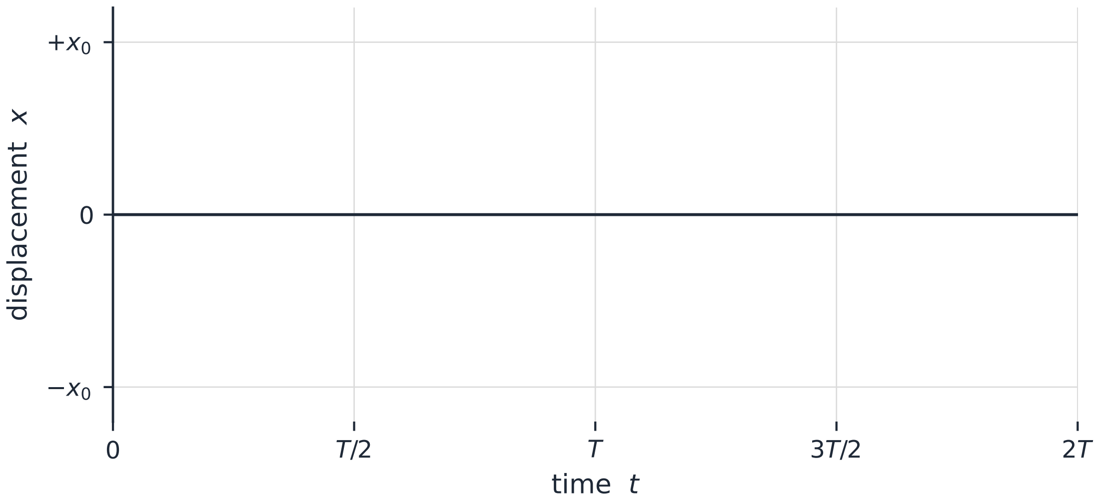
(a) A second graph plots the ball's **acceleration** against its **displacement** and gives a **straight line through the origin with a negative gradient**. **Explain** how this shows that the motion is simple harmonic. [2]

(b) **Determine** the period T and the angular frequency ω of the oscillations. [3]

(c) The magnitude of the gradient of that acceleration-displacement line is 61 s^{-2}. **Show that** this is consistent with your value of ω. [2]

(d) **State** what is meant by **damping**. [2]

(e) The ball is now made to oscillate in a beaker of oil, and is released from the same displacement x_0. On the axes below, **sketch** a possible variation of its displacement with time from $t = 0$ to $t = 2T$. [3]



Solutions

2.1 Acceleration is **proportional to displacement** from a fixed point, and **directed towards** that point (opposite to displacement).

2.2 The **maximum displacement** from the equilibrium position.

2.3 $v_{\text{max}} = \omega x_0 = 5.0 \times 0.040 = 0.20$ m s^{-1}.

2.4 They are **antiphase** (phase difference π / 180°).

3.1 **B** — proportional to displacement and opposite in direction.

3.2 (a) $\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{0.50} = 12.6$ rad s^{-1}. (b) $v_{\text{max}} = \omega x_0 = 12.6 \times 0.030 = 0.38$ m s^{-1}. (c) $a_{\text{max}} = \omega^2 x_0 = 12.6^2 \times 0.030 = 4.7$ m s^{-2}.

3.3 $v = \omega\sqrt{x_0^2 - x^2} = 8.0\sqrt{0.10^2 - 0.060^2} = 8.0\sqrt{0.0064} = 8.0 \times 0.080 = 0.64$ m s^{-1}.

3.4 Displacement is a **sine** (or cosine) curve; acceleration is the **same shape inverted** (negative); they are **antiphase** (180° out of phase).

3.5 (a) The line is **straight and through the origin**, so a is **proportional to** x; its **negative gradient** shows a is in the **opposite direction** to x — i.e. $a = -\omega^2 x$, the SHM condition.

3.5 (b) gradient $= \dfrac{-8.0}{0.05} = -160$ s^{-2} $= -\omega^2$; $\omega = \sqrt{160} = 12.6$ rad s^{-1}; $T = \dfrac{2\pi}{\omega} = 0.50$ s.

3.6 (a) A straight line through the origin shows $a \propto x$; the negative gradient shows the acceleration is always directed **opposite** to the displacement, i.e. towards the equilibrium position — which is the definition of simple harmonic motion.
(b) $T = \dfrac{6.4}{8} = 0.80$ s; $\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{0.80} = 7.9$ rad s^{-1}.
(c) For SHM $a = -\omega^2 x$, so the magnitude of the gradient is ω^2: $\omega = \sqrt{61} = 7.8$ rad s^{-1}, which agrees with (b).
(d) Damping is the loss of energy from an oscillating system to its surroundings, usually through **resistive forces**, so the **amplitude decreases** with time.
(e) A curve starting at $+x_0$, oscillating with the **same period** T so that it still completes two cycles by $2T$, with each maximum displacement **smaller** than the one before, decaying smoothly.