A-LEVEL PHYSICS • EXERCISE SHEET

15.3 Kinetic theory of gases

Name: ______________ Class: __________ Date: ____________ **Total: 23 marks**

KEY WORDS root-mean-square 均方根 • kinetic theory 分子动理论 • kinetic energy 动能 • translational kinetic en
• mean-square speed 均方速率

Objective

Build the skills to answer exam questions on the **kinetic theory of gases**.

You must be able to:

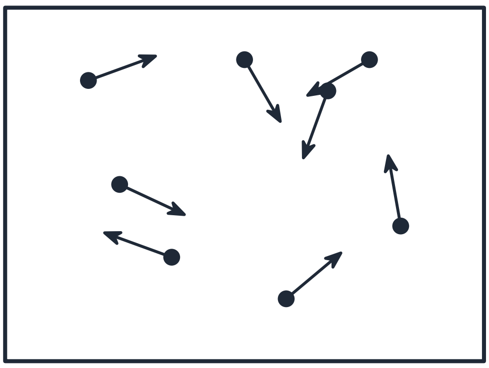
- state the **assumptions** of the kinetic theory and explain gas pressure
- use $pV = \frac{1}{3}Nm\langle c^2\rangle$ and the **r.m.s. speed**
- use mean translational KE $= \frac{3}{2}kT$

1 Worked examples

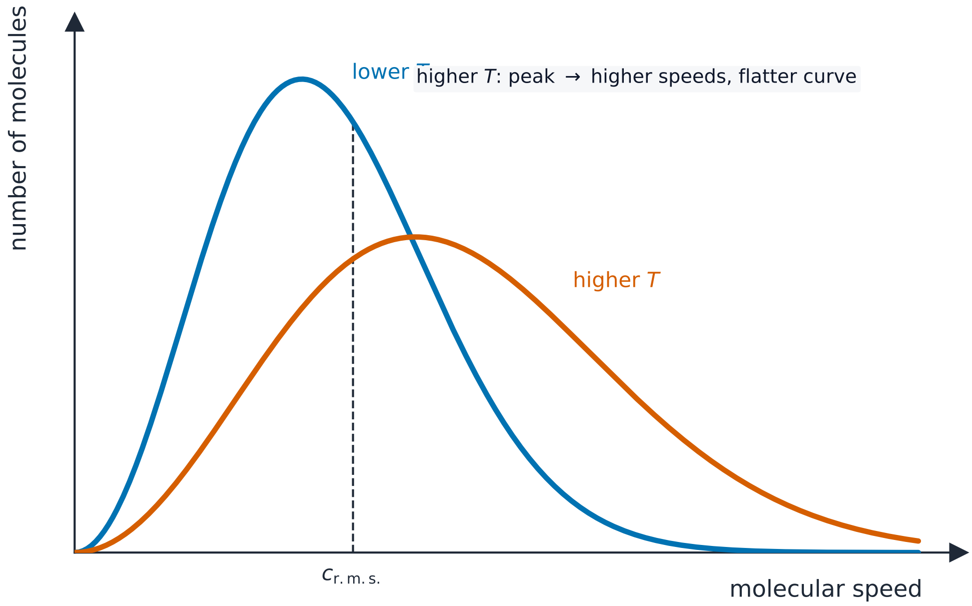
Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Kinetic model

A gas is many identical molecules in **continuous random motion**; their collisions with the walls cause **pressure**:



random motion — wall collisions cause pressure



The Maxwell-Boltzmann speed distribution

Assumptions: many molecules, random motion; their volume is negligible; collision time is negligible; no forces except in collisions; collisions are **elastic**; Newton's laws hold.

■ Key results

$$pV = \tfrac{1}{3}Nm\langle c^2\rangle, \qquad c_{\mathrm{r.m.s.}} = \sqrt{\langle c^2\rangle}.$$

Comparing with $pV = NkT$ gives the **mean translational KE** of a molecule:

$$\tfrac{1}{2}m\langle c^2\rangle = \tfrac{3}{2}kT.$$

So molecular KE depends only on **temperature**.

2 Practice

Now apply the methods above.

2.1 State **two** assumptions of the kinetic theory of gases. [2]

2.2 Explain how gas molecules cause **pressure** on the walls of a container. [2]

2.3 State what is meant by the **root-mean-square (r.m.s.) speed**. [1]

2.4 Write the relationship $pV = \frac{1}{3}Nm\langle c^2\rangle$ in words. [1]

3 Exam-style questions

3.1 The **mean translational kinetic energy** of a gas molecule is: [1]

- **A** kT
- **B** $\frac{3}{2}kT$
- **C** $\frac{1}{2}mv^2$ only
- **D** RT

3.2 Calculate the mean translational kinetic energy of a gas molecule at 300 K. ($k = 1.38 \times 10^{-23}$.) [2]

3.3 A gas at 300 K has molecules of mass 5.3×10^{-26} kg. Calculate the **r.m.s. speed**. ($k = 1.38 \times 10^{-23}$.) [3]

3.4 Explain, using the kinetic theory, why the pressure of a fixed mass of gas **increases** when it is heated at **constant volume**. [2]

3.5 A fixed sample of an ideal gas at thermodynamic temperature T has internal energy U.

(a) **State** what is meant by the **internal energy** of a system. [2]

(b) **Explain** why the internal energy of an **ideal** gas is directly proportional to its thermodynamic temperature. [2]

(c) The gas is first **compressed**, which raises its temperature to $3T$; work W is done on it during this compression. It is then **cooled at constant volume** back to $2T$. **Complete** the table, giving each entry in terms of some or all of W and U. [4]

process	**work done on the gas**	**thermal energy supplied to the gas**	**increase in internal energy**
compression to $3T$	W	____________	____________
cooling to $2T$	____________	____________	____________

(d) **Explain** why the work done on the gas during the second process is what it is. [1]

Solutions

2.1 Any two: many identical molecules in **random motion**; molecular **volume negligible**; **no forces** except during collisions; collisions are **elastic**; collision time negligible.

2.2 Molecules collide with the walls and **change momentum** (rebound); this exerts a **force** on the wall; force over the wall area is the **pressure**.

2.3 The **square root of the mean of the squared speeds** of the molecules ($\sqrt{\langle c^2 \rangle}$).

2.4 The product of pressure and volume equals **one third** of (the number of molecules $\times$ molecular mass $\times$ mean-square speed).

3.1 **B** — $\frac{3}{2}kT$.

3.2 $E = \frac{3}{2}kT = 1.5 \times (1.38 \times 10^{-23}) \times 300 = 6.2 \times 10^{-21}$ J.

3.3 $\frac{1}{2}m\langle c^2 \rangle = \frac{3}{2}kT \Rightarrow \langle c^2 \rangle = \dfrac{3kT}{m}$; $c_{\text{r.m.s.}} = \sqrt{\dfrac{3(1.38 \times 10^{-23})(300)}{5.3 \times 10^{-26}}} = 4.8 \times 10^2$ m s^{-1}.

3.4 Heating raises the **mean KE / mean speed** of the molecules; they hit the walls **harder and more often**, so the pressure rises (volume fixed).

3.5 (a) The **sum of the random kinetic and potential energies** of all the molecules, where "random" means the motion has no overall direction — the whole container moving does not count.
(b) In an ideal gas the molecules exert **no forces** on one another except during collisions, so the total **potential** energy is zero and the internal energy is entirely kinetic. The mean kinetic energy of a molecule is $\frac{3}{2}kT$, i.e. proportional to T, so the total is proportional to T as well.
(c) Internal energy is proportional to T, so at T it is U, at $3T$ it is $3U$ and at $2T$ it is $2U$.

process	**work done on the gas**	**thermal energy supplied**	**increase in internal energy**
compression to $3T$	W	$2U - W$	$+2U$
cooling to $2T$	0	$-U$	$-U$

(d) At **constant volume** the gas neither expands nor is compressed, so no work is done on or by it.