

13.4 Gravitational potential

Name: _____ Class: _____ Date: _____

Total: 16 marks

KEY WORDS gravitational potential 引力势 • gravitational potential energy 重力势能 • mass 质量
 • point mass 质点 • test mass 检验质量

Objective

Build the skills to answer exam questions on **gravitational potential** — $\phi = -GM/r$ and gravitational potential energy.

You must be able to:

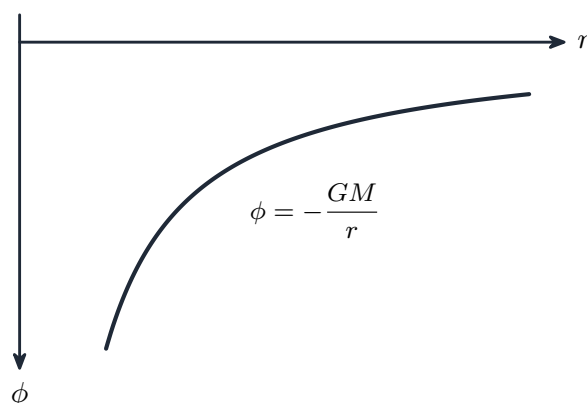
- define **gravitational potential** and use $\phi = -\frac{GM}{r}$
- use $E_P = -\frac{GMm}{r}$ for two point masses
- explain why potential and E_P are negative

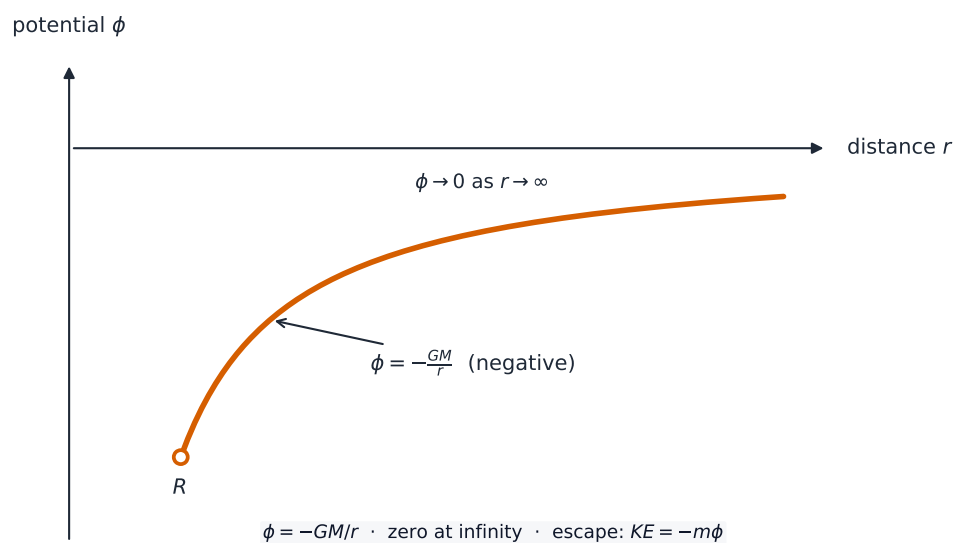
1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Gravitational potential

Gravitational potential ϕ at a point is the **work done per unit mass** to bring a small test mass **from infinity** to that point. Potential is **zero at infinity** and **negative** everywhere else:





The gravitational potential well

$$\phi = -\frac{GM}{r} \quad (\text{J kg}^{-1}), \quad E_P = m\phi = -\frac{GMm}{r}.$$

For two radii, $\Delta E_P = GMm \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$.

2 Practice

Now apply the methods above.

2.1 Define gravitational potential.

[1]

2.2 State why gravitational potential is negative.

[1]

2.3 State the unit of gravitational potential.

[1]

2.4 Write the equation for the gravitational potential energy of two point masses.[1]

3 Exam-style questions

3.1 The gravitational potential at **infinity** is:

[1]

- **A** a maximum positive value
 - **B** zero
 - **C** a large negative value
 - **D** infinite
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3.2 Calculate the gravitational potential at the Earth's surface. ($M = 6.0 \times 10^{24}$ kg, $R = 6.4 \times 10^6$ m, $G = 6.67 \times 10^{-11}$.)

[3]

3.3 Calculate the gravitational potential energy of a 1200 kg satellite at $r = 7.0 \times 10^6$ m from the Earth's centre.

[2]

3.4 Explain why **two closer** masses have **more negative** gravitational potential energy.

[2]

3.5 A 1000 kg satellite orbiting a planet is moved from an orbit of radius 7.0×10^6 m to one of radius 1.4×10^7 m. Its gravitational potential energy **increases** by 2.86×10^{10} J. Calculate the **mass of the planet**, and give a **unit**. ($G = 6.67 \times 10^{-11}$.)

[4]

Solutions

2.1 The **work done per unit mass** in bringing a small test mass from **infinity** to that point.

2.2 Gravity is **attractive**, so it does the work as the mass moves in from infinity (where $\phi = 0$); the potential therefore becomes **negative**.

2.3 J kg^{-1} .

2.4 $E_{\text{P}} = -\frac{GMm}{r}$.

3.1 B — zero.

3.2 $\phi = -\frac{GM}{R} = -\frac{(6.67 \times 10^{-11})(6.0 \times 10^{24})}{6.4 \times 10^6} = -6.3 \times 10^7 \text{ J kg}^{-1}$.

3.3 $E_{\text{P}} = -\frac{GMm}{r} = -\frac{(6.67 \times 10^{-11})(6.0 \times 10^{24})(1200)}{7.0 \times 10^6} = -6.9 \times 10^{10} \text{ J}$.

3.4 As r decreases, $-\frac{GMm}{r}$ becomes **more negative**; the masses are more **tightly bound** — more work would be needed to separate them to infinity.

3.5 $\Delta E_{\text{P}} = GMm \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$; $\frac{1}{r_1} - \frac{1}{r_2} = \frac{1}{7.0 \times 10^6} - \frac{1}{1.4 \times 10^7} = 7.14 \times 10^{-8} \text{ m}^{-1}$;

$M = \frac{\Delta E_{\text{P}}}{Gm \left(\frac{1}{r_1} - \frac{1}{r_2} \right)} = \frac{2.86 \times 10^{10}}{(6.67 \times 10^{-11})(1000)(7.14 \times 10^{-8})} = 6.0 \times 10^{24} \text{ kg}$.