13.3 Gravitational field of a point mass

Name: ______________ Class: __________ Date: ____________ **Total: 26 marks**

KEY WORDS gravitational field 重力场 • gravitational field strength 重力场强度 • field line 场线 • newton's law of • mass 质量

Objective

Build the skills to answer exam questions on the **gravitational field of a point mass** — the equation $g = GM/r^2$.

You must be able to:

- derive and use $g = \dfrac{GM}{r^2}$
- describe how g varies with distance
- explain why g is nearly constant near the Earth's surface

1 Worked examples

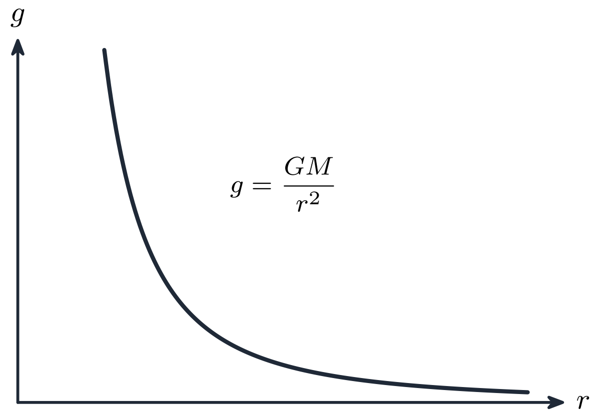
Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

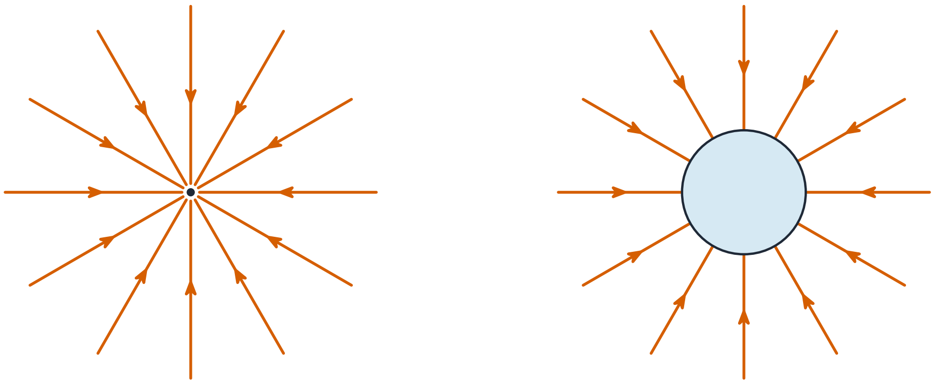
Field of a point mass

Put $F = \dfrac{GMm}{r^2}$ into $g = F/m$:

$$g = \frac{GM}{r^2}.$$

So g falls off as an **inverse square** of distance:





point mass

uniform sphere

Field lines around a point mass

■ Nearly constant near the surface

Rising a height $h \ll R$ changes r from R to $R + h$, but $(R + h)/R \approx 1$, so g barely changes — it is effectively constant in a lab.

2 Practice

Now apply the methods above.

2.1 Write the equation for the gravitational field strength g due to a point mass. [1]

2.2 State how g varies with distance r from a point mass. [1]

2.3 At the Earth's surface $g = 9.8$ N kg^{-1}. Find g at a distance of $2R$ from the centre (a height R above the surface). [2]

2.4 Explain why g is **nearly constant** near the Earth's surface. [1]

3 Exam-style questions

3.1 At **twice** the distance from a point mass, g becomes: [1]

- **A** twice as large
- **B** half as large
- **C** one quarter as large
- **D** four times as large

3.2 The Moon has mass 7.3×10^{22} kg and radius 1.7×10^{6} m. Calculate g at its surface. ($G = 6.67 \times 10^{-11}$.) [3]

3.3 Derive $g = \dfrac{GM}{r^2}$ from Newton's law of gravitation and $g = F/m$. [2]

3.4 A planet has the same mass as Earth but **twice** the radius. State its surface g as a fraction of Earth's. [1]

3.5 On the axes, **sketch** how the gravitational field strength g of a planet varies with distance r from its centre, for r from the surface ($r = R$) out to $r = 4R$. Mark the surface value g_0. [2]

3.6 A planet of mass M and radius R has a gravitational field strength of g_0 at its surface. A satellite of mass m orbits it in a circle of radius r, where $r > R$.

(a) **State** why the gravitational field strength inside a **uniform** sphere behaves differently from the field outside it. [1]

(b) **Show that** the orbital speed of the satellite is $v = \sqrt{\dfrac{GM}{r}}$, and hence that it does **not** depend on the mass of the satellite. [3]

(c) **Derive** an expression for the period T of the orbit in terms of G, M and r, and state the name of the law it expresses. [3]

(d) A geostationary satellite orbits the Earth once every 24 hours. Taking $GM = 4.00 \times 10^{14}\ \mathrm{m^3\,s^{-2}}$, **determine** the radius of its orbit. [3]

(e) **State** two further conditions a geostationary orbit must satisfy, beyond having the right period. [2]

Solutions

2.1 $g = \dfrac{GM}{r^2}$.

2.2 $g \propto \dfrac{1}{r^2}$ (inverse-square).

2.3 At $2R$, g is $\dfrac{1}{2^2} = \dfrac{1}{4}$ of the surface value: $g = 9.8/4 = 2.45\ \text{N kg}^{-1}$.

2.4 A small height change is **tiny compared with the Earth's radius**, so r (and hence $g \propto 1/r^2$) barely changes.

3.1 **C** — one quarter.

3.2 $g = \dfrac{GM}{R^2} = \dfrac{(6.67 \times 10^{-11})(7.3 \times 10^{22})}{(1.7 \times 10^6)^2} = 1.7\ \text{N kg}^{-1}$.

3.3 Force on a test mass m: $F = \dfrac{GMm}{r^2}$; field strength $g = \dfrac{F}{m} = \dfrac{GM}{r^2}$.

3.4 $\dfrac{1}{4}$ of Earth's (since $g \propto 1/R^2$).

3.5 A curve **starting at** g_0 at $r = R$ and **falling** as an inverse square — to $g_0/4$ at $2R$, $g_0/9$ at $3R$, $g_0/16$ at $4R$ — approaching (but never reaching) zero.

3.6 (a) Outside the sphere all its mass may be treated as concentrated at the centre, giving $g \propto \dfrac{1}{x^2}$; inside, only the mass **closer to the centre** than the point contributes, so the field falls towards zero at the centre instead of rising.
(b) The gravitational force provides the centripetal force: $\dfrac{GMm}{r^2} = \dfrac{mv^2}{r}$. The satellite mass m appears on both sides and **cancels**, leaving $v^2 = \dfrac{GM}{r}$, so $v = \sqrt{\dfrac{GM}{r}}$.
(c) $T = \dfrac{2\pi r}{v} = 2\pi r\sqrt{\dfrac{r}{GM}}$, so $T^2 = \dfrac{4\pi^2 r^3}{GM}$ — that is $T^2 \propto r^3$, **Kepler's third law**.
(d) $T = 24 \times 3600 = 86\,400$ s; $r^3 = \dfrac{GMT^2}{4\pi^2} = \dfrac{4.00 \times 10^{14} \times (86\,400)^2}{4\pi^2} = 7.56 \times 10^{22}$; $r = 4.2 \times 10^7$ m.
(e) It must orbit **above the equator**, in the plane of the equator, and travel **west to east**, the same direction as the Earth's rotation.