13.1 Gravitational field

Name: ____________ Class: ________ Date: __________ **Total: 20 marks**

KEY WORDS gravitational field 重力场 • field line 场线 • gravitational field strength 重力场强度 • force 力 • mass 质量

Objective

Build the skills to answer exam questions on the **gravitational field** — its definition and field lines.

You must be able to:

- define **gravitational field strength** as force per unit mass
- represent a gravitational field with **field lines**

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

Field strength

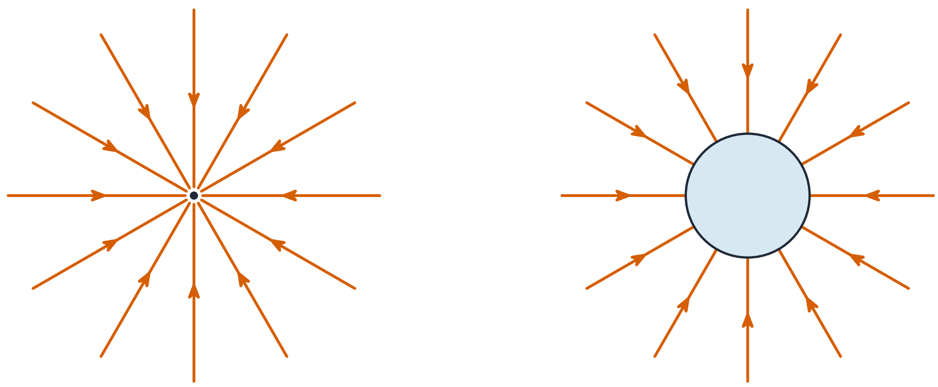
A **gravitational field** is a region where a mass feels a force. The **field strength** is the force per unit mass on a small test mass:

$$g = \frac{F}{m} \quad (\mathrm{N\ kg^{-1}}).$$

It is a **vector**, pointing towards the source mass (the same value as the free-fall acceleration).

Field lines

(a) radial field (b) uniform field



point mass uniform sphere

The radial gravitational field of a sphere

- around a **point mass / sphere**: **radial**, pointing **inwards**;
- near a surface (small region): **uniform** — parallel, equally spaced, pointing down.

Closer lines → stronger field.

2 Practice

Now apply the methods above.

2.1 Define **gravitational field strength**. [1]

2.2 State the unit of gravitational field strength. [1]

2.3 State what a gravitational **field line** represents. [1]

2.4 State the shape of the gravitational field around a **point mass**. [1]

3 Exam-style questions

3.1 Gravitational field strength is equal to: [1]

- **A** force $\times$ mass
- **B** force $\div$ mass
- **C** mass $\div$ force
- **D** mass $\times$ acceleration

3.2 A mass of 2.0 kg experiences a gravitational force of 19.6 N. Find the **gravitational field strength**. [2]

3.3 Explain what **closer** field lines indicate. [1]

3.4 Sketch the gravitational field lines (a) around a point mass and (b) near a flat surface. [2]

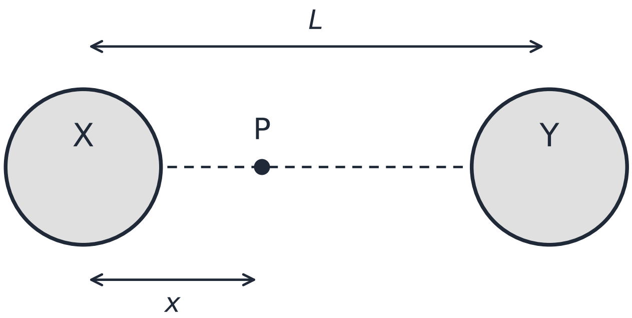
3.5 A point mass M sits at the origin. Point P is a distance x from it.

(a) **Define** the **gravitational field** at a point. [1]

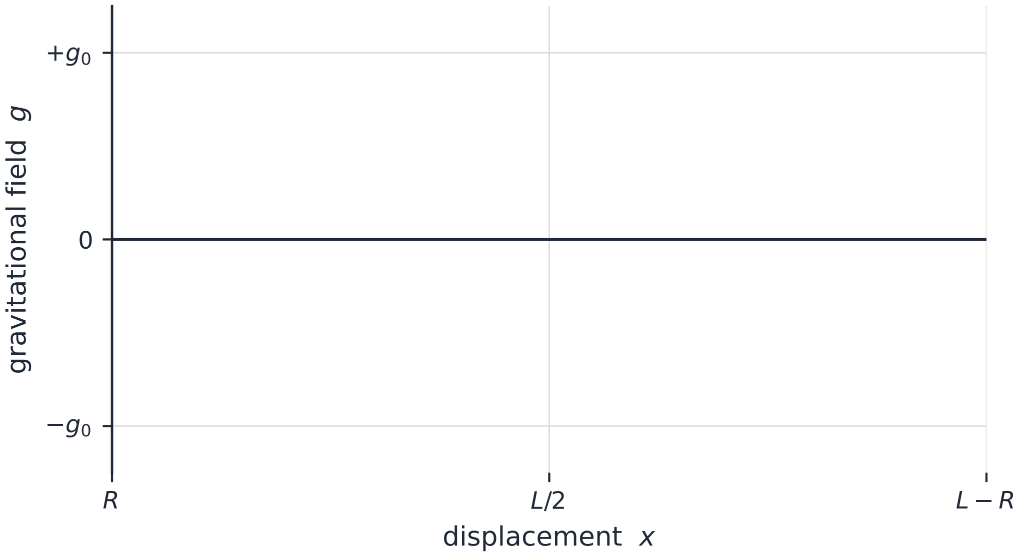
(b) By considering the force on a test mass m placed at P, **derive** an expression for the gravitational field strength g at P in terms of M and x. State the meaning of any other symbol you use. [2]

(c) Point Q lies on the **opposite** side of the mass from P, at a distance $\frac{x}{2}$ from it. **Compare** the gravitational field at Q with the field at P. [2]

(d) Two identical uniform spheres X and Y, each of radius R, have their centres a distance L apart. Point P now lies on the line joining their centres, a displacement x from the centre of X. The gravitational field strength at the surface of each sphere is g_0.



On the axes below, **sketch** how the field g at P varies with x, from $x = R$ to $x = L - R$. Take the direction from X towards Y as positive. [3]



(e) **Explain** why the field is zero at the midpoint, even though neither sphere has been removed. [2]

Solutions

2.1 The **gravitational force per unit mass** on a small test mass at that point ($g = F/m$).

2.2 N kg^{-1}.

2.3 The **direction of the gravitational force** on a (test) mass at that point.

2.4 **Radial**, pointing **inwards** towards the mass.

3.1 **B** — force ÷ mass.

3.2 $g = F/m = 19.6/2.0 = 9.8$ N kg^{-1}.

3.3 A **stronger** gravitational field.

3.4 (a) **radial** lines pointing inward to the mass; (b) **parallel, equally spaced** lines pointing down towards the surface.

3.5 (a) The gravitational **force per unit mass** acting on a small test mass placed at that point.
(b) The force on the test mass is $F = \dfrac{GMm}{x^2}$, where G is the gravitational constant; the field strength is the force per unit mass, so $g = \dfrac{F}{m} = \dfrac{GM}{x^2}$.
(c) The fields point in **opposite directions**, since each points towards the mass; and because $g \propto \dfrac{1}{x^2}$, halving the distance makes the field at Q **four times** the field at P.
(d) The curve starts at $(R,\ -g_0)$, since at X's surface the field points back towards X, i.e. negative; it passes through zero at the midpoint $x = L/2$; and it ends at $(L - R,\ +g_0)$, becoming shallower up to the midpoint and steeper after it.
(e) The two spheres are identical and the midpoint is equidistant from both, so their fields there are **equal in magnitude** and **opposite in direction**, and since field strength is a vector the two cancel to zero. (The gravitational *potential* there is not zero — only the field is.)