

12.1 Kinematics of uniform circular motion

Name: _____ Class: _____ Date: _____

Total: 14 marks

KEY WORDS angular speed 角速度 • radian 弧度 • period 周期 • arc 弧 • revolution 圈

Objective

Build the skills to answer exam questions on **circular motion kinematics** — radians, angular speed, $v = r\omega$, and **connected** wheels.

You must be able to:

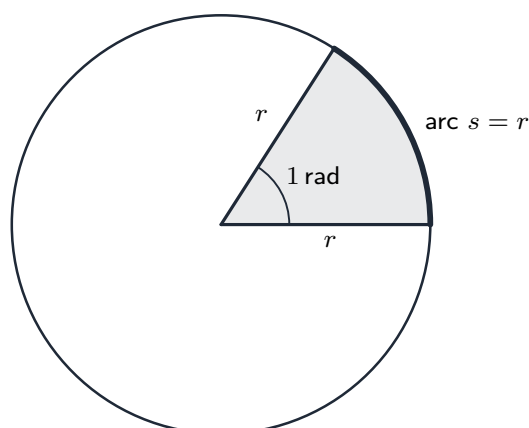
- define the **radian** and convert between radians and degrees
- use **angular speed** $\omega = \theta/t = 2\pi/T = 2\pi f$ and $v = r\omega$
- handle **connected wheels** (same axle $\rightarrow$ same ω ; linked by a belt/chain $\rightarrow$ same rim speed)

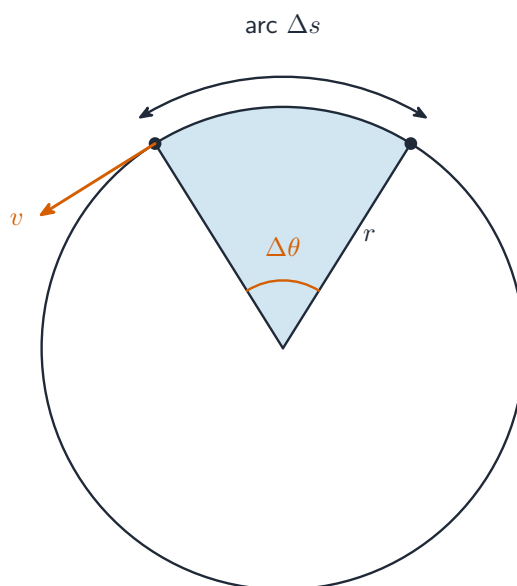
1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Radians

One **radian** is the angle whose arc length equals the radius: $\theta = s/r$:





Angular speed in uniform circular motion

A full circle = 2π rad; $1 \text{ rad} = 180^\circ/\pi \approx 57.3^\circ$. **Use radian mode** on your calculator.

■ Angular speed

$$\omega = \frac{\theta}{t} = \frac{2\pi}{T} = 2\pi f \quad (\text{rad s}^{-1}), \quad v = r\omega.$$

Worked example. A point on the equator (radius $6.4 \times 10^6 \text{ m}$, period 24 h):

$$\omega = \frac{2\pi}{24 \times 3600} = 7.3 \times 10^{-5} \text{ rad s}^{-1}; \quad v = r\omega = 465 \text{ m s}^{-1}.$$

■ Connected wheels

Exam questions often link wheels, cogs or pulleys. Two rules:

- **Fixed on the same axle** → they turn with the **same** ω (but different rim speeds if the radii differ).
- **Linked by a belt or chain** (no slipping) → the **rim speeds are equal**: $r_1\omega_1 = r_2\omega_2$.

Worked example. A belt links pulley A (radius 0.10 m, $\omega_A = 30 \text{ rad s}^{-1}$) to pulley B (radius 0.050 m). Rim speed $v = r_A\omega_A = 0.10 \times 30 = 3.0 \text{ m s}^{-1}$; so $\omega_B = \frac{v}{r_B} = \frac{3.0}{0.050} = 60 \text{ rad s}^{-1}$ (the smaller pulley spins faster).

2 Practice

Now apply the methods above.

2.1 Define the **radian**. [1]

2.2 Convert 90° to **radians**. [1]

2.3 A wheel turns at 2.0 revolutions per second. Find its **angular speed**. [2]

2.4 Two pulleys are linked by a belt. State what is **equal** for the two pulleys. [1]

3 Exam-style questions

3.1 A full circle, in radians, is: [1]

- **A** π
 - **B** 2π
 - **C** 360
 - **D** 1
-

3.2 A car travels around a circular track of radius 50 m at 20 m s^{-1} . Find its **angular speed**. [2]

3.3 In a workshop, a motor turns a pulley of radius 25 mm at 1500 revolutions per minute. A tight belt runs from that pulley to a larger pulley of radius 75 mm on the shaft of a fan.

(a) Determine the angular speed of the **motor pulley**, in rad s^{-1} . [2]

(b) The belt does not slip. Determine the angular speed of the **fan**, and state what stays the same for both pulleys. [3]

3.4 Convert 1.5 rad to **degrees**. [1]

Solutions

2.1 The angle subtended at the centre of a circle by an arc whose **length equals the radius**.

2.2 $90^\circ = \frac{\pi}{2} \text{ rad} \approx 1.57 \text{ rad}.$

2.3 $\omega = 2\pi f = 2\pi \times 2.0 = 4\pi = 12.6 \text{ rad s}^{-1}.$

2.4 Their **rim speeds** ($v = r\omega$) are equal (the belt does not slip).

3.1 **B** — 2π .

3.2 $\omega = v/r = 20/50 = 0.40 \text{ rad s}^{-1}.$

3.3 (a) 1500 revolutions per minute = 25 revolutions per second, and one revolution is $2\pi \text{ rad}$, so $\omega = 25 \times 2\pi = 157 \text{ rad s}^{-1}.$

3.3 (b) The belt gives both pulleys the same **rim speed** $v = r\omega$: $v = 0.025 \times 157 = 3.93 \text{ m s}^{-1}$, so the fan turns at $\omega = \frac{3.93}{0.075} = 52 \text{ rad s}^{-1}$ — a third of the motor's, because its radius is three times larger.

3.4 $1.5 \times \frac{180}{\pi} = 86^\circ.$