

10.2 Kirchhoff's laws

Name: _____ Class: _____ Date: _____

Total: 19 marks

KEY WORDS kirchhoff's first law 基尔霍夫第一定律 • kirchhoff's second law 基尔霍夫第二定律 • resistance 电阻
• internal resistance 内阻 • parallel 并联

Objective

Build the skills to answer exam questions on **Kirchhoff's laws** and analysing **combined** circuits.

You must be able to:

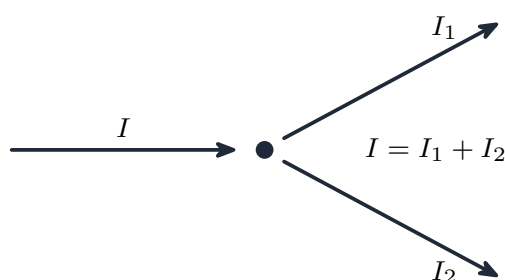
- state both **Kirchhoff's laws** and the conservation law each follows from
- combine resistors in **series** and **parallel**
- analyse a **combined** circuit: total resistance, total current, then each **branch** current and p.d.

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Kirchhoff's laws

First law (junction): current in = current out — from conservation of **charge**:



Second law (loop): around any closed loop, the **total e.m.f.** = **total p.d.** — from conservation of **energy**.

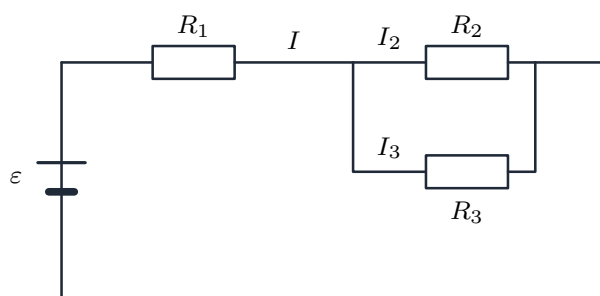
■ Combining resistors

- **Series** (same current): $R = R_1 + R_2 + \dots$
- **Parallel** (same p.d.): $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \dots$

Worked example. $3.0\ \Omega$ and $6.0\ \Omega$ in parallel: $\frac{1}{R} = \frac{1}{3.0} + \frac{1}{6.0} = \frac{1}{2.0}$, so $R = 2.0\ \Omega$.

■ Analysing a combined circuit

Real exam circuits mix series and parallel. Work in **steps**: collapse the parallel part, find the total current, then split it.



Worked example. $\varepsilon = 12\ \text{V}$ (negligible internal resistance), $R_1 = 4.0\ \Omega$ in series with $R_2 = 6.0\ \Omega$ and $R_3 = 3.0\ \Omega$ in parallel.

1. Parallel pair: $\frac{1}{R_p} = \frac{1}{6.0} + \frac{1}{3.0} = \frac{1}{2.0}$, so $R_p = 2.0\ \Omega$.
2. Total: $R = R_1 + R_p = 4.0 + 2.0 = 6.0\ \Omega$; current $I = \frac{\varepsilon}{R} = \frac{12}{6.0} = 2.0\ \text{A}$.
3. P.d. across the parallel part: $V_p = IR_p = 2.0 \times 2.0 = 4.0\ \text{V}$.
4. Branch currents: $I_2 = \frac{V_p}{R_2} = \frac{4.0}{6.0} = 0.67\ \text{A}$, $I_3 = \frac{V_p}{R_3} = \frac{4.0}{3.0} = 1.33\ \text{A}$ (check: $0.67 + 1.33 = 2.0\ \text{A}$ ✓).

2 Practice

Now apply the methods above.

2.1 State **Kirchhoff's first law** and which conservation law it follows from. [2]

2.2 State **Kirchhoff's second law** and which conservation law it follows from. [2]

2.3 Find the combined resistance of $4.0\ \Omega$ and $6.0\ \Omega$ in **series**. [1]

2.4 Find the combined resistance of two $4.0\ \Omega$ resistors in **parallel**. [2]

3 Exam-style questions

3.1 Three $6.0\ \Omega$ resistors in **parallel** have a combined resistance of: [1]

- **A** $2.0\ \Omega$
- **B** $6.0\ \Omega$
- **C** $18\ \Omega$
- **D** $0.50\ \Omega$

3.2 A $12\ \text{V}$ battery drives current through a $3.0\ \Omega$ and a $6.0\ \Omega$ resistor in **series**. Calculate the current and the p.d. across each resistor. [4]

3.3 A battery of e.m.f. 6.0 V and negligible internal resistance is connected to $R_1 = 2.0\ \Omega$ in series with two resistors $R_2 = 4.0\ \Omega$ and $R_3 = 4.0\ \Omega$ in **parallel** (as in the worked example).

(a) Calculate the total resistance and the current from the battery. [3]

(b) Calculate the current in **each** of R_2 and R_3 . [3]

3.4 At a junction, 5.0 A enters and one branch carries 2.0 A away. Find the current in the other branch. [1]

Solutions

2.1 The **total current into a junction equals the total current out**; it follows from conservation of **charge**.

2.2 Around any closed loop, the **total e.m.f. equals the total p.d.**; it follows from conservation of **energy**.

2.3 $R = 4.0 + 6.0 = 10\ \Omega$.

2.4 $\frac{1}{R} = \frac{1}{4.0} + \frac{1}{4.0} = \frac{1}{2.0}$, so $R = 2.0\ \Omega$.

3.1 $A = 6.0/3 = 2.0\ \Omega$.

3.2 $R = 3.0 + 6.0 = 9.0\ \Omega$; $I = 12/9.0 = 1.33\ A$; $V_3 = IR = 1.33 \times 3.0 = 4.0\ V$, $V_6 = 1.33 \times 6.0 = 8.0\ V$.

3.3 (a) Parallel pair: $\frac{1}{R_p} = \frac{1}{4.0} + \frac{1}{4.0} = \frac{1}{2.0}$, so $R_p = 2.0\ \Omega$; total $R = 2.0 + 2.0 = 4.0\ \Omega$; $I = 6.0/4.0 = 1.5\ A$.

3.3 (b) $V_p = IR_p = 1.5 \times 2.0 = 3.0\ V$; each branch $I = \frac{3.0}{4.0} = 0.75\ A$, and the two are equal because $R_2 = R_3$.

3.4 $5.0 - 2.0 = 3.0\ A$.