1.2 SI units

Name: ____________ Class: ________ Date: __________ **Total: 27 marks**

KEY WORDS current 电流 • temperature 温度 • derived unit 导出单位 • prefix 词头 • si units 国际单位制

Objective

Build the skills to answer exam questions on **SI units 国际单位制** — base units, derived units, homogeneity and prefixes.

You must be able to:

- recall the **five SI base units** (kg, m, s, A, K) and complete an exam table of them
- express a **derived unit** in SI base units, and find the units of an unknown constant
- check the **homogeneity** of an equation, and use it to find an unknown **power**
- convert a **compound unit** that carries **prefixes**

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

Base and derived units

The **base quantities**: mass (kg), length (m), time (s), current (A), temperature (K). Build a **derived unit** from the equation that defines it:

$$\begin{array}{ccccc} \text{force} & = & \text{mass} & \times & \text{acceleration} \\ \downarrow & & \downarrow & & \downarrow \\ \text{kg m s}^{-2} & = & \text{kg} & \times & \text{m s}^{-2} \end{array} \qquad \textit{so } 1\ N = 1\ kg\ m\ s^{-2}$$

So 1 N = 1 kg m s^{-2}; the **joule** = kg m^2 s^{-2}; the **watt** = kg m^2 s^{-3}; the **pascal** = kg m^{-1} s^{-2}.

Units of an unknown constant

Rearrange for the constant, then substitute base units. For $F = kAv$, where A is an area and v a speed:

$$k = \frac{F}{Av} = \frac{\text{kg m s}^{-2}}{\text{m}^2 \times \text{m s}^{-1}} = \text{kg m}^{-2}\,\text{s}^{-1}$$

Write the fraction first, then cancel one line at a time — that first line is the method mark.

Checking homogeneity

An equation is **homogeneous** when **every term** has the **same base units**. Check $v^2 = u^2 + 2as$:

- LHS: $(\mathrm{m\ s^{-1}})^2 = \mathrm{m^2\,s^{-2}}$
- u^2: $\mathrm{m^2\,s^{-2}}$; $2as$: $(\mathrm{m\ s^{-2}})(\mathrm{m}) = \mathrm{m^2\,s^{-2}}$

All three are $\mathrm{m^2\,s^{-2}}$ — homogeneous. Matching units do **not** prove an equation fully correct (a wrong number, like the 2, would still match), but **mismatched** units prove it wrong. Two terms that are **added** must have the same units, so $F = kv^2 + m$ can never be right.

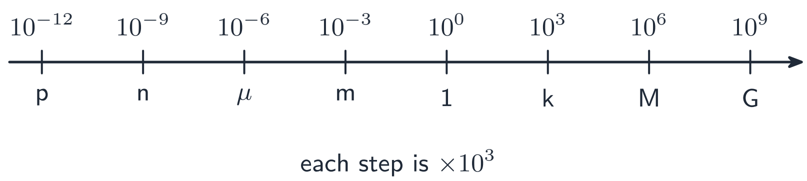
Finding an unknown power

If the equation is homogeneous, the powers must balance. In $T = 2\pi\left(\frac{L}{g}\right)^n$, with T a time, L a length and g an acceleration:

$$\left[\frac{L}{g}\right] = \frac{\mathrm{m}}{\mathrm{m\ s^{-2}}} = \mathrm{s}^2 \quad\Rightarrow\quad (\mathrm{s}^2)^n = \mathrm{s}^1 \quad\Rightarrow\quad 2n = 1 \quad\Rightarrow\quad n = \tfrac{1}{2}$$

The 2π has no units, so it never enters the check.

Prefixes, and compound units



p	n	μ	m	c	d	k	M	G	T
10^{-12}	10^{-9}	10^{-6}	10^{-3}	10^{-2}	10^{-1}	10^{3}	10^{6}	10^{9}	10^{12}

To convert a **compound** unit, replace **each** prefix by its power of ten — and remember a prefix inside a power is raised to that power too:

$$0.25\ \mathrm{kN\ mm^{-2}} = 0.25 \times \frac{10^3\ \mathrm{N}}{(10^{-3}\ \mathrm{m})^2} = 0.25 \times \frac{10^3}{10^{-6}}\ \mathrm{N\ m^{-2}} = 2.5 \times 10^8\ \mathrm{N\ m^{-2}}$$

$(10^{-3})^2 = 10^{-6}$, not 10^{-3} — that squaring is where most marks are lost.

2 Practice

Now apply the methods above.

2.1 Name the **five** SI base quantities and give each unit. [2]

2.2 Write the SI **base units** of **force**, and of **pressure**. [2]

2.3 Write 5 mA in **amperes** and 2.0 GHz in **hertz**, both in standard form. [2]

2.4 Use $W = Fs$ to express the **joule** in SI base units. [2]

2.5 A density is 3.0 g cm^{-3}. Express it in kg m^{-3}. [2]

2.6 In the equation $P = k\rho v^3$, P is a power, ρ a density and v a speed. **Determine** the SI base units of k. [2]

3 Exam-style questions

3.1 Which of these is **not** an SI base unit? [1]

- **A** ampere
- **B** coulomb
- **C** kelvin
- **D** kilogram

3.2 Which quantity has the same SI base units as **force**? [1]

- **A** $\dfrac{\text{energy}}{\text{distance}}$
- **B** $\dfrac{\text{energy}}{\text{time}}$
- **C** momentum × distance
- **D** momentum × time

3.3 What is 6.0 kN cm^{-2} expressed in N m^{-2}? [1]

- **A** 6.0×10^1
- **B** 6.0×10^5
- **C** 6.0×10^7
- **D** 6.0×10^9

3.4 The speed v of a wave on the surface of a liquid is given by $v = k\left(\dfrac{p}{\rho}\right)^n$, where p is a pressure, ρ is a density and k has no units. The equation is homogeneous. What is the value of n? [1]

- **A** -2
- **B** $-\frac{1}{2}$
- **C** $\frac{1}{2}$
- **D** 2

3.5 An engineering team releases an unmanned **submersible** (an underwater craft) from rest at the surface of a lake. Its volume is 0.25 m^3 and its weight is 2.1 kN. The density of the lake water is 1030 kg m^{-3}.

(a) The team records three quantities during the dive. **Complete** the table by writing the SI base units of each quantity, and ticking whether it is a scalar or a vector. [3]

quantity	SI base units	scalar	vector
water temperature	______________	☐	☐
water pressure	______________	☐	☐
drag force on the submersible	______________	☐	☐

(b) **Show that** the upthrust on the fully submerged submersible is about 2.5 kN. (Upthrust = weight of the water displaced.) [2]

(c) **Determine** the magnitude and the direction of the initial acceleration of the submersible when it is released. [3]

(d) The drag force F on the submersible is modelled as $F = kv^2$, where v is its speed. **Determine** the SI base units of k. [2]

(e) A student writes the drag equation as $F = kv^2 + m$, where m is the mass of the submersible. Use **units** to explain why this equation cannot be correct. [1]

Solutions

2.1 mass — kg; length — m; time — s; current — A; temperature — K.

2.2 force: kg m s^{-2}; pressure: kg m^{-1} s^{-2}.

2.3 5 mA $= 5 \times 10^{-3}$ A; 2.0 GHz $= 2.0 \times 10^9$ Hz.

2.4 $W = Fs$, so J = (kg m s^{-2})(m) = kg m^2 s^{-2}.

2.5 $3.0\ \dfrac{\text{g}}{\text{cm}^3} = 3.0 \times \dfrac{10^{-3}\ \text{kg}}{(10^{-2}\ \text{m})^3} = 3.0 \times \dfrac{10^{-3}}{10^{-6}} = 3.0 \times 10^3\ \text{kg m}^{-3}$.

2.6 $k = \dfrac{P}{\rho v^3} = \dfrac{\text{kg m}^2\,\text{s}^{-3}}{(\text{kg m}^{-3})(\text{m}^3\,\text{s}^{-3})} = \dfrac{\text{kg m}^2\,\text{s}^{-3}}{\text{kg s}^{-3}} = \text{m}^2$.

3.1 B — the coulomb is a derived unit (1 C = 1 A s).

3.2 A — $\dfrac{\text{energy}}{\text{distance}} = \dfrac{\text{kg m}^2\,\text{s}^{-2}}{\text{m}} = \text{kg m s}^{-2}$, the units of force. (B is power; C is kg m^2 s^{-1}; D is kg m.)

3.3 C — $6.0 \times \dfrac{10^3}{(10^{-2})^2} = 6.0 \times \dfrac{10^3}{10^{-4}} = 6.0 \times 10^7\ \text{N m}^{-2}$.

3.4 C — $\left[\dfrac{p}{\rho}\right] = \dfrac{\text{kg m}^{-1}\,\text{s}^{-2}}{\text{kg m}^{-3}} = \text{m}^2\,\text{s}^{-2}$, so $(\text{m}^2\,\text{s}^{-2})^n = \text{m s}^{-1}$ gives $2n = 1$ and $n = \frac{1}{2}$.

3.5 (a) water temperature — K, scalar; water pressure — kg m^{-1} s^{-2}, scalar; drag force — kg m s^{-2}, vector.
(b) mass of water displaced $= \rho V = 1030 \times 0.25 = 257.5$ kg; upthrust $= \rho V g = 1030 \times 0.25 \times 9.81 = 2526\ \text{N} \approx 2.5$ kN.
(c) resultant force $= 2526 - 2100 = 426$ N upwards; mass $= \dfrac{2100}{9.81} = 214$ kg; $a = \dfrac{426}{214} = 2.0\ \text{m s}^{-2}$, directed **upwards**.
(d) $k = \dfrac{F}{v^2} = \dfrac{\text{kg m s}^{-2}}{\text{m}^2\,\text{s}^{-2}} = \text{kg m}^{-1}$.
(e) kv^2 has the units of force, kg m s^{-2}, but m has the unit kg. Terms that are **added** must have the same units, so the equation is not homogeneous and cannot be correct.