Exercise walk-through

Polarisation ▪ 7.5

eXercise

You must be able to

- explain that only **transverse 横波** waves can be polarised
- use **Malus's law** $I = I_0 \cos^2 \theta$, and work **backwards** to find the angle
- sketch how the transmitted intensity varies as a filter is **rotated**
- link transmitted **intensity** to **amplitude 振幅** using $I \propto A^2$

Method — Polarisation

Polarisation makes a transverse wave oscillate in **one plane** only. Only **transverse** waves can be polarised (their oscillation is perpendicular to the travel direction, so different planes exist). **Longitudinal 纵波** waves such as sound **cannot** — so polarisation is a **test for a transverse wave**.

Method — Malus's law

Plane-polarised light of intensity I_0 meets a filter whose **transmission axis 透射轴** is at angle θ to the plane of polarisation. The transmitted intensity is:

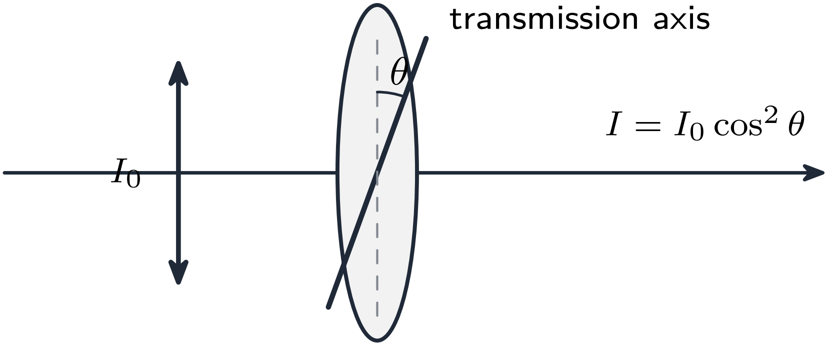
$$I = I_0 \cos^2 \theta.$$

- $\theta = 0°$: $I = I_0$ (all passes); $\theta = 90°$: $I = 0$ (blocked); $\theta = 45°$: $I = \frac{1}{2} I_0$; $\theta = 60°$: $I = \frac{1}{4} I_0$.

Worked example (forward). $I_0 = 12$ W m^{-2}, $\theta = 30°$: $I = 12 \cos^2 30° = 12 \times 0.75 = 9.0$ W m^{-2}.

Worked example (backwards). For what θ is the intensity **halved**? Set $\cos^2 \theta = 0.5$, so $\cos \theta = \sqrt{0.5}$ and $\theta = 45°$.

Method — Malus's law

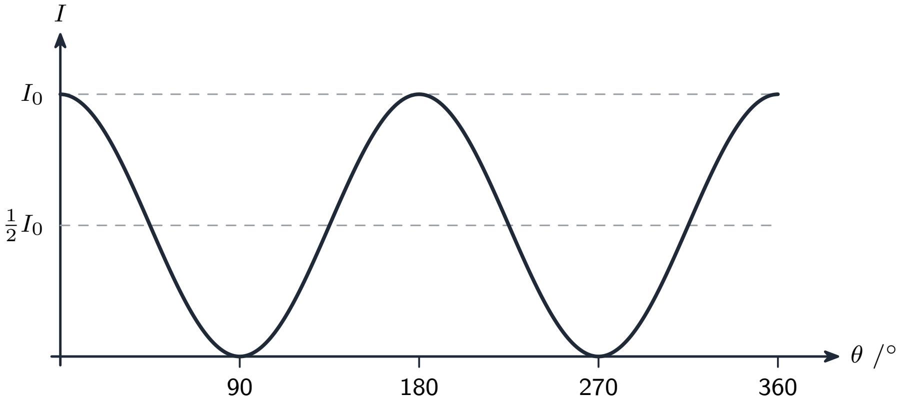


Method — Rotating the filter

If the angle θ between the axis and the polarisation is changed (by **rotating** the filter), the transmitted intensity traces a $\cos^2$ curve — maximum I_0 at $0°, 180°, 360°$ and zero at $90°, 270°$:

The "starts perpendicular" case. If the axis **begins perpendicular** to the polarisation and is then rotated by an angle ϕ, the angle in the formula is $\theta = 90° - \phi$. So $I = I_0 \cos^2(90° - \phi) = I_0 \sin^2 \phi$: the intensity **starts at zero**, rises to I_0 after $\phi = 90°$, and so on.

Method — Rotating the filter



Method — Intensity and amplitude

Intensity is proportional to amplitude squared, $I \propto A^2$, so for a wave through a filter:

$$\frac{A}{A_0} = \sqrt{\frac{I}{I_0}}.$$

Worked example. If a filter transmits $I = 0.25\, I_0$, the amplitude is $A = A_0\sqrt{0.25} = 0.50\, A_0$ — the amplitude falls to **half**, even though the intensity falls to a **quarter**.

Method — Two filters in series

Apply Malus's law **once per filter**; the second angle is measured from the **first** filter's axis. Two **crossed** filters (90° apart) transmit **nothing**.

Practice 2.1 ▪ 1 mark

State what **polarisation** means.

Solution 2.1

Worked steps

Restricting the oscillations of a transverse wave to **one plane** B1.

Practice 2.2 ▪ 1 mark

State why **sound** cannot be polarised.

Solution 2.2

Method: Polarisation

Worked steps

Sound is **longitudinal** — its oscillation is along the direction of travel, so there is no other plane to restrict it to **B1**.

Practice 2.3 ▪ 1 mark

Plane-polarised light meets a filter whose axis is at 60° to its plane. State the **fraction** of intensity transmitted.

Solution 2.3

Method: Malus's law

Worked steps

$\cos^2 60° = 0.25$, i.e. **one quarter** B1.

Practice 2.4 ▪ 1 mark

A filter transmits **half** the incident intensity. State the angle θ between its axis and the plane of polarisation.

Solution 2.4

Method: Rotating the filter

Worked steps

$\cos^2\theta = 0.5 \Rightarrow \cos\theta = \sqrt{0.5} \Rightarrow \theta = 45^\circ$ **B1**.

Practice 2.5 ▪ 1 mark

A filter transmits $I = 0.25\, I_0$. State the transmitted amplitude as a fraction of A_0.

Solution 2.5

Method: Intensity and amplitude

Worked steps

$A = A_0\sqrt{0.25} = 0.50\,A_0$ — **half** the amplitude **B1**.

Exam-style 3.1 ▪ 1 mark

Which of these **can** be polarised?

A sound waves

B all electromagnetic waves

C longitudinal waves

D all of these

Solution 3.1

A sound waves

B all electromagnetic waves ✓

C longitudinal waves

D all of these

Worked steps

B — all electromagnetic waves (they are transverse).

Exam-style 3.2 ▪ 2 marks

Plane-polarised light of intensity 12 W m^{-2} meets a filter whose axis is at $30°$ to the plane of polarisation. Calculate the transmitted intensity.

Solution 3.2

Method: Malus's law

Worked steps

$I = I_0 \cos^2\theta = 12 \times \cos^2 30° = 12 \times 0.75 = 9.0\ \mathrm{W\ m^{-2}}$ **M1** **A1**.

Exam-style 3.3 ▪ 3 marks

Plane-polarised light passes through **two** filters. The first axis is at 30° to the plane of polarisation; the second is a further 30° beyond the first. The light reaching the first filter has intensity 8.0 W m^{-2}. Calculate the intensity emerging from the **second** filter.

Solution 3.3

Method: Malus's law

Worked steps

First filter: $I_1 = 8.0\cos^2 30° = 8.0 \times 0.75 = 6.0$ W m^{-2} **M1**. Second filter (a further 30°): $I_2 = I_1 \cos^2 30° = 6.0 \times 0.75 = 4.5$ W m^{-2} **M1** **A1**.

Exam-style 3.4 ▪ 2 marks

A photographer looks at a shop window through a polarising filter. The light reflected from the glass is plane-polarised, of intensity I_0, and the filter's transmission axis starts **parallel** to the plane of polarisation. The filter is then rotated through $360°$ about the viewing direction.

(a) On the axes, sketch how the transmitted intensity I varies with the rotation angle ϕ.

(b) Determine the smallest rotation ϕ that **halves** the transmitted intensity.

(c) At that angle, find the transmitted **amplitude** as a fraction of the incident amplitude A_0, and explain why the photographer rotates the filter to cut the glare.

Solution 3.4

Method: Malus's law

Worked steps

(c) Intensity $\propto$ amplitude^2, so $A = A_0\sqrt{0.5} = 0.71\,A_0$ **M1** **A1**. Glare from the glass is polarised in one plane, so turning the axis towards $90°$ to that plane cuts the reflected light while the unpolarised light from inside the shop is only halved **B1**.

Exam-style 3.5 ▪ 2 marks

Explain why the ability to **polarise** a wave shows that the wave is **transverse**.

Solution 3.5

Method: Polarisation

Worked steps

Polarisation restricts the oscillation to **one plane** **B1**; this is only possible if the oscillation is **perpendicular** to the direction of travel — i.e. the wave is transverse (a longitudinal wave oscillates along the travel direction, so it has only one possible plane) **B1**.