Exercise walk-through

Stress and strain ▪ 6.1

eXercise

You must be able to

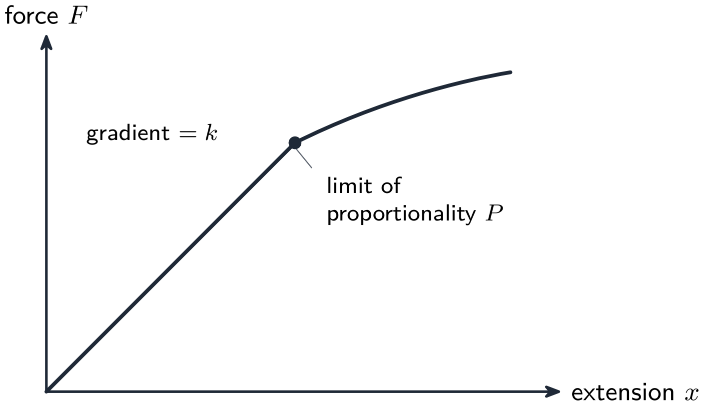
- use **Hooke's law** $F = kx$ and find k from a graph
- define and use **stress**, **strain** and the **Young modulus**
- describe the experiment to measure the **Young modulus** of a wire

Method — Hooke's law and spring constant

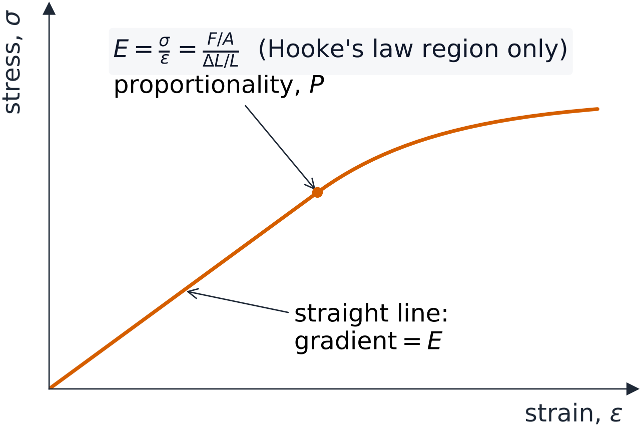
For small loads, **extension is proportional to load**: $F = kx$, so the **spring constant** $k = F/x$ (unit N m^{-1}). On a force–extension graph the **gradient is** k in the straight region, up to the **limit of proportionality**:

Worked example. A spring extends 4.0 cm under a 2.0 N load:
$k = F/x = 2.0/0.040 = 50$ N m^{-1}.

Method — Hooke's law and spring constant



Method — Hooke's law and spring constant



The stress-strain graph for a material

Method — Stress, strain, Young modulus

$$\text{stress } \sigma = \frac{F}{A} \text{ (Pa)}, \qquad \text{strain } \varepsilon = \frac{x}{L} \text{ (no unit)}, \qquad E = \frac{\sigma}{\varepsilon} \text{ (Pa)}.$$

Young modulus experiment (wire): measure the original length L and diameter ($\rightarrow$ area A); add known loads; measure the extension x (e.g. with a marker and ruler/travelling microscope); plot stress against strain; E is the gradient of the straight part.

Practice 2.1 ▪ 1 mark

State **Hooke's law**.

Solution 2.1

Worked steps

The **extension is proportional to the load** (up to the limit of proportionality) B1.

Practice 2.2 ▪ 2 marks

A spring extends 4.0 cm when a 2.0 N load is hung from it. Find the **spring constant**.

Solution 2.2

Method: Hooke's law and spring constant

Worked steps

$k = F/x = 2.0/0.040 = 50\ \mathrm{N\ m}^{-1}$ M1 A1.

Practice 2.3 ▪ 2 marks

Define **stress** and **strain**.

Solution 2.3

Worked steps

Stress = force per unit cross-sectional area (F/A); **strain** = extension per unit original length (x/L) **B1** **B1**.

Practice 2.4 ▪ 1 mark

State the unit of the **Young modulus**.

Solution 2.4

Method: Stress, strain, Young modulus

Worked steps

Pa (N m^{-2}) **B1**.

Exam-style 3.1 ▪ 1 mark

In the straight region, the **gradient** of a force–extension graph gives the:

- **A** Young modulus
- **B** spring constant
- **C** stress
- **D** strain

Solution 3.1

Method: Hooke's law and spring constant

A Young modulus

B spring constant ✓

C stress

D strain

Worked steps

B — spring constant.

Exam-style 3.2 ▪ 4 marks

A wire of length 2.0 m and cross-sectional area 1.0×10^{-6} m^2 stretches by 1.0 mm under a 100 N load. Calculate (a) the stress, (b) the strain, (c) the Young modulus.

Solution 3.2

Method: Stress, strain, Young modulus

Worked steps

(a) $\sigma = F/A = 100/(1.0 \times 10^{-6}) = 1.0 \times 10^{8}$ Pa **M1** **A1**. (b) $\varepsilon = x/L = (1.0 \times 10^{-3})/2.0 = 5.0 \times 10^{-4}$ **A1**. (c) $E = \sigma/\varepsilon = (1.0 \times 10^{8})/(5.0 \times 10^{-4}) = 2.0 \times 10^{11}$ Pa **A1**.

Exam-style 3.3 ▪ 4 marks

Describe an experiment to measure the **Young modulus** of a metal wire.

Solution 3.3

Method: Stress, strain, Young modulus

Worked steps

Any four: measure original length L and diameter ($\rightarrow$ area A) **B1**; hang the wire and add known loads **B1**; measure the extension with a marker against a fixed scale **B1**; plot stress against strain and find E from the gradient of the straight part **B1**.

Exam-style 3.4 ▪ 1 mark

Explain what is meant by the **limit of proportionality**.

Solution 3.4

Method: Hooke's law and spring constant

Worked steps

The point beyond which **extension is no longer proportional to load** (the F–x line stops being straight) **B1**.

Exam-style 3.5 ▪ 1 mark

A wire has length L, cross-sectional area A, Young modulus E and resistivity ρ.
(a) **Define** the **Young modulus** of a material.
(b) **State** an expression, in terms of some or all of L, A, E and ρ, for the resistance R_0 of the wire.
(c) **Show that** the spring constant k_0 of the wire is given by $k_0 = \dfrac{EA}{L}$.
(d) The wire is now stretched by a steadily increasing force F, staying within its limit of proportionality. In the space below, **sketch** two graphs: the variation with F of the **resistance** R of the wire, and the variation with F of its **spring constant** k. **Label** each graph and its axes.
(e) A copper wire of diameter 1.6 mm has a resistance of $0.034\ \Omega$. The resistivity of copper is $1.7 \times 10^{-8}\ \Omega\,\mathrm{m}$. **Show that** the length of the wire is about 3.8 m.

Exam-style 3.5 ▪ 1 mark

(f) The Young modulus of copper is 1.3×10^{11} Pa. **Determine** the spring constant of this wire.

Solution 3.5

Method: Hooke's law and spring constant

Worked steps

(a) The **ratio of stress to strain** (for a material below its limit of proportionality) **B1**.

(b) $R_0 = \dfrac{\rho L}{A}$ **B1**.

(c) The spring constant is $k = \dfrac{F}{x}$, and the Young modulus is $E = \dfrac{\text{stress}}{\text{strain}} = \dfrac{F/A}{x/L} = \dfrac{FL}{Ax}$ **M1**. Rearranging, $E = \dfrac{F}{x} \cdot \dfrac{L}{A} = \dfrac{k_0 L}{A}$, so $k_0 = \dfrac{EA}{L}$ **A1**.

(d) **Resistance:** a straight line of **positive** gradient starting at $(0, R_0)$ — stretching

Solution 3.5

makes the wire longer and thinner, so R rises **B1**. **Spring constant:** a **horizontal** line starting at $(0, k_0)$ — within the limit of proportionality k is a constant of the wire **B1**. Both axes labelled, each line labelled **B1**.

(e) $A = \pi r^2 = \pi(0.80 \times 10^{-3})^2 = 2.01 \times 10^{-6}$ m^2 **M1**;

$L = \dfrac{R_0 A}{\rho} = \dfrac{0.034 \times 2.01 \times 10^{-6}}{1.7 \times 10^{-8}} = 3.8$ m **A1**. (The **radius** is half the diameter — the commonest slip here.)

(f) $k_0 = \dfrac{EA}{L} = \dfrac{1.3 \times 10^{11} \times 2.01 \times 10^{-6}}{3.8}$ **M1** $= 6.9 \times 10^4$ N m^{-1} **A1**.