Exercise walk-through

Density and pressure ▪ 4.3

eXercise

You must be able to

- use **density** $\rho = m/V$ and **pressure** $p = F/A$
- derive and use **hydrostatic pressure** $\Delta p = \rho g \Delta h$
- find **upthrust** with $F = \rho g V$ (Archimedes' principle)
- get a density from **measured** side lengths, and its **percentage uncertainty**
- decide whether an object **floats**, and how much of it sits below the surface

Method — Density and pressure

$$\rho = \frac{m}{V}\ (\text{kg m}^{-3}), \qquad p = \frac{F}{A}\ (\text{Pa} = \text{N m}^{-2}).$$

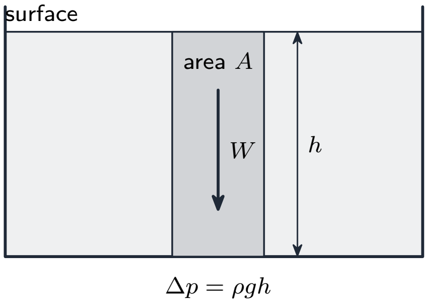
Water has $\rho = 1000$ kg m^{-3}.

Method — Hydrostatic pressure

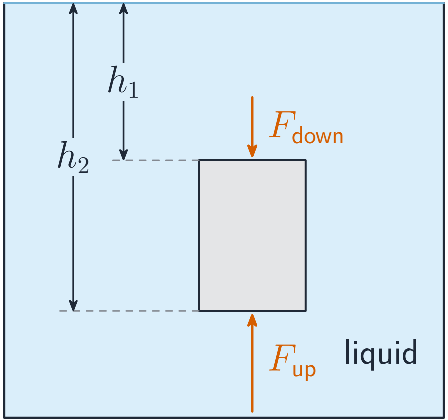
A column of liquid (area A, depth h) has weight $W = \rho A h g$ pressing on its base, so the **extra** pressure is

$$\Delta p = \frac{W}{A} = \rho g h.$$

It depends only on **depth and density** — not on the container's shape. (For total pressure, add atmospheric pressure $\approx 1.0 \times 10^5$ Pa.)



Method — Hydrostatic pressure



Upthrust arises from the pressure difference with depth

Method — Upthrust

A submerged object feels a larger pressure on its **bottom** than its top — this pressure difference gives an upward **upthrust**:

$$F = \rho g V,$$

where V is the volume of fluid displaced (Archimedes' principle).

Method — Density from measurements, with its uncertainty

An exam density is rarely handed to you: it comes from a mass and three measured sides, each with an uncertainty.

Worked example. $m = (0.243 \pm 0.001)$ kg; $x = (5.41 \pm 0.01)$ cm, $y = (11.09 \pm 0.01)$ cm, $z = (1.62 \pm 0.01)$ cm.

$$V = xyz = 5.41 \times 11.09 \times 1.62 = 97.2 \text{ cm}^3 = 9.72 \times 10^{-5} \text{ m}^3$$

$$\rho = \frac{m}{V} = \frac{0.243}{9.72 \times 10^{-5}} = 2500 \text{ kg m}^{-3}$$

For the uncertainty, add the **percentage** uncertainties of everything multiplied or divided:

Method — Density from measurements, with its uncertainty

$$\frac{0.001}{0.243} + \frac{0.01}{5.41} + \frac{0.01}{11.09} + \frac{0.01}{1.62} = 0.41 + 0.18 + 0.09 + 0.62 = 1.3\%$$

The **smallest** side carries the largest percentage — that is the measurement worth improving, and examiners ask exactly that.

Method — Floating

An object floats when the upthrust can balance its weight **before** it is fully under:

$$\rho_{\text{object}}\, V_{\text{total}}\, g = \rho_{\text{fluid}}\, V_{\text{submerged}}\, g \quad \Rightarrow \quad \frac{V_{\text{submerged}}}{V_{\text{total}}} = \frac{\rho_{\text{object}}}{\rho_{\text{fluid}}}$$

So ice (920 kg m^{-3}) floats in water with 92% of its volume below the surface. If $\rho_{\text{object}} > \rho_{\text{fluid}}$ the fraction would exceed 1, which is the algebra telling you it sinks.

Practice 2.1 ▪ 1 mark

Define **density** and give its SI unit.

Solution 2.1

Worked steps

Density is **mass per unit volume**, unit kg m^{-3} **B1**.

Practice 2.2 ▪ 2 marks

A block has mass 2400 kg and volume 0.30 m^3. Find its **density**.

Solution 2.2

Method: Density from measurements, with its uncertainty

Worked steps

$\rho = m/V = 2400/0.30 = 8000\ \text{kg m}^{-3}$ M1 A1.

Practice 2.3 ▪ 2 marks

A force of 200 N acts on an area of 0.50 m^2. Find the **pressure**.

Solution 2.3

Method: Hydrostatic pressure

Worked steps

$p = F/A = 200/0.50 = 400$ Pa M1 A1.

Practice 2.4 ▪ 1 mark

State what causes the **upthrust** on a submerged object.

Solution 2.4

Worked steps

The **difference in pressure** between the bottom and top of the object (the bottom is deeper, so at higher pressure) B1.

Exam-style 3.1 ▪ 1 mark

The extra pressure at the bottom of a liquid column depends on:

- **A** the cross-sectional area only
- **B** the depth and the density
- **C** the shape of the container
- **D** the total volume of liquid

Solution 3.1

Method: Hydrostatic pressure

A the cross-sectional area only

B the depth and the density ✓

C the shape of the container

D the total volume of liquid

Worked steps

B — the depth and the density.

Exam-style 3.2 ▪ 2 marks

Calculate the extra pressure due to the water at the bottom of a pool 2.5 m deep. ($\rho = 1000$ kg m^{-3}, $g = 9.81$ m s^{-2}.)

Solution 3.2

Worked steps

$\Delta p = \rho g h = 1000 \times 9.81 \times 2.5 = 2.5 \times 10^4$ Pa **M1** **A1**.

Exam-style 3.3 ▪ 2 marks

A block of volume 2.0×10^{-3} m^3 is fully submerged in water ($\rho = 1000$ kg m^{-3}). Calculate the **upthrust** on it.

Solution 3.3

Method: Floating

Worked steps

$F = \rho g V = 1000 \times 9.81 \times 2.0 \times 10^{-3} = 19.6\ \mathrm{N}$ **M1** **A1**.

Exam-style 3.4 ▪ 3 marks

Derive the hydrostatic pressure equation $\Delta p = \rho g h$ from the definitions of **pressure** and **density**.

Solution 3.4

Method: Hydrostatic pressure

Worked steps

The column's mass $m = \rho V = \rho A h$ **M1**; its weight $W = mg = \rho A h g$ presses on area A **M1**; so $\Delta p = W/A = \rho g h$ **A1**.

Exam-style 3.5 ▪ 1 mark

A student measures a rectangular metal block to find the density of the metal. The results are

quantity	**measurement**
mass m	(0.512 ± 0.001) kg
length x	(8.02 ± 0.01) cm
width y	(4.15 ± 0.01) cm
height z	(2.05 ± 0.01) cm

Exam-style 3.5 ▪ 1 mark

(a) **Define** density.
(b) **Determine** the density of the metal.
(c) **Determine** the percentage uncertainty in the density.
(d) **State**, with a reason, which single measurement the student should improve first.
(e) The block is lowered into water ($\rho = 1000$ kg m^{-3}) until it is fully submerged. **Show that** the upthrust on it is about 0.67 N, and **explain** why the block still sinks.
(f) A block of wood of density 650 kg m^{-3} floats in the same water. **Determine** the fraction of its volume that lies **below** the surface.
(g) A swimming pool is 25 m long, 10 m wide and 2.0 m deep. **Estimate** the force that the water exerts on the bottom of the pool, and state one assumption you make.

Solution 3.5

Method: Floating

Worked steps

(a) Density is **mass per unit volume** B1.

(b) $V = 8.02 \times 4.15 \times 2.05 = 68.2\ \text{cm}^3 = 6.82 \times 10^{-5}\ \text{m}^3$ M1 M1;

$\rho = \dfrac{0.512}{6.82 \times 10^{-5}} = 7.5 \times 10^3\ \text{kg m}^{-3}$ A1.

(c) $\dfrac{0.001}{0.512} = 0.20\%$; $\dfrac{0.01}{8.02} = 0.12\%$; $\dfrac{0.01}{4.15} = 0.24\%$; $\dfrac{0.01}{2.05} = 0.49\%$ M1 M1; total $= 0.20 + 0.12 + 0.24 + 0.49 = 1.1\%$ A1.

(d) The **height** z B1 — it is the smallest length, so the same ± 0.01 cm is the largest percentage of it and it dominates the total B1.

Solution 3.5

(e) $F = \rho_{\text{water}} g V = 1000 \times 9.81 \times 6.82 \times 10^{-5} = 0.67$ N **M1** **A1**. The block's own weight is $mg = 0.512 \times 9.81 = 5.0$ N, far greater than the upthrust, so the resultant force is downwards and it sinks — its density is over seven times that of water **B1**.

(f) Floating means upthrust = weight, so $\rho_{\text{water}} V_{\text{sub}} g = \rho_{\text{wood}} V_{\text{total}} g$ **M1**;

$\dfrac{V_{\text{sub}}}{V_{\text{total}}} = \dfrac{650}{1000} = 0.65$, i.e. 65% below the surface **A1**.

(g) Pressure due to the water at the bottom:

$\Delta p = \rho g h = 1000 \times 9.81 \times 2.0 = 1.96 \times 10^4$ Pa **M1**; area $= 25 \times 10 = 250$ m^2, so $F = pA = 1.96 \times 10^4 \times 250 = 4.9 \times 10^6$ N **A1**. Assumption: the depth is the same everywhere (a flat bottom), and atmospheric pressure is not included — adding it would roughly double the force **B1**.