Exercise walk-through

Turning effects of forces ▪ 4.1

eXercise

You must be able to

- use the **centre of gravity 重心** when drawing weight
- define and calculate the **moment 力矩** of a force
- define a **couple** and calculate its **torque 力偶矩**
- locate the **centre of gravity** of a hanging object, and use a torque to find how far it lies from the pivot

Method — Centre of gravity and moment

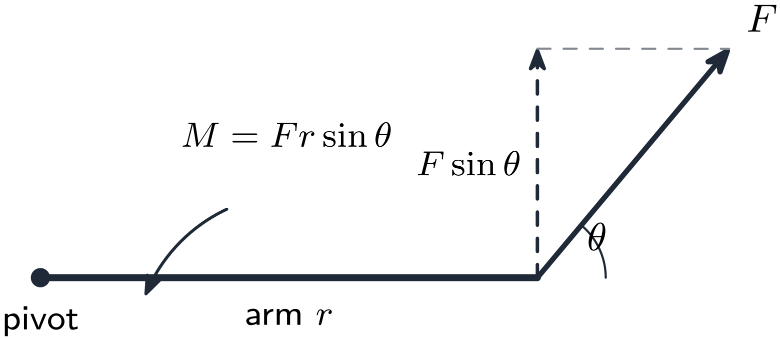
The whole **weight** acts at the **centre of gravity** (the middle of a uniform shape). The **moment** of a force about a point is

$$M = F \times d,$$

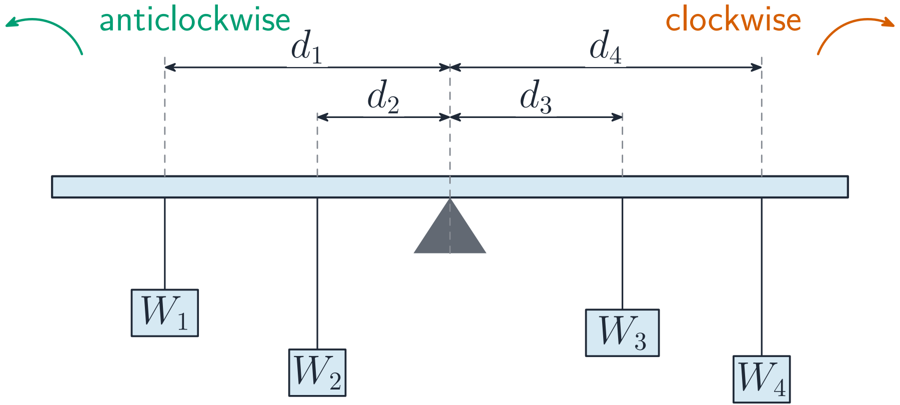
where d is the **perpendicular distance** from the point to the force's line of action. If the force acts at angle θ to a lever arm of length r, then $M = Fr\sin\theta$ — only the part **perpendicular** to the arm turns it:
Worked example. 50 N at the end of a 0.40 m spanner, at $30°$ to it:

$$M = 50 \times 0.40 \times \sin 30° = 10 \text{ N m}.$$

Method — Centre of gravity and moment



Method — Centre of gravity and moment



The principle of moments balances turning effects

Method — Finding a centre of gravity

For a **non-uniform** object the centre of gravity is not at the middle, and the exam asks you to find it two ways.

Hang it up. An object free to swing settles with its centre of gravity **vertically below** the point it hangs from. So the centre of gravity lies **somewhere on that vertical line** — hang it from a second point and the two lines cross at it.

Hold it still. If the object is held so that its centre of gravity is a horizontal distance d from the pivot, the weight has a moment mgd about the pivot, and the torque you must apply is equal and opposite:

$$\tau = mgd \quad \Rightarrow \quad d = \frac{\tau}{mg}$$

Method — Finding a centre of gravity

Worked example. Holding a 2.0 kg non-uniform plate on a rod takes a torque of 1.6 N m:

$$d = \frac{1.6}{2.0 \times 9.81} = 0.082 \text{ m}$$

The direction tells you which side: if the centre of gravity is to the **right** of the rod, the weight turns the plate **clockwise**, so the applied torque must be **anticlockwise**.

Method — Couple and torque

A **couple** is two forces that are **equal, opposite**, and a perpendicular distance apart — it produces **turning only** (zero resultant force). Its **torque** is

$$\tau = F \times d,$$

where d is the distance between the two lines of action. *(Two equal forces in the* ***same*** *direction are* ***not*** *a couple.)*

Practice 2.1 ▪ 1 mark

Define the **moment** of a force.

Solution 2.1

Worked steps

Force $\times$ the **perpendicular distance** from the pivot to the line of action of the force B1.

Practice 2.2 ▪ 2 marks

A 20 N force acts at right angles to a lever, 0.30 m from the pivot. Find its **moment**.

Solution 2.2

Method: Centre of gravity and moment

Worked steps

$M = Fd = 20 \times 0.30 = 6.0$ N m M1 A1.

Practice 2.3 ▪ 2 marks

State the **two** conditions a pair of forces must meet to be a **couple**.

Solution 2.3

Method: Couple and torque

Worked steps

Equal in size and **opposite** in direction B1; their lines of action are a **perpendicular distance apart** (not the same line) B1.

Practice 2.4 ▪ 1 mark

State where the **weight** of a uniform rod acts.

Solution 2.4

Method: Centre of gravity and moment

Worked steps

At its **centre of gravity** — the midpoint of the uniform rod B1.

Exam-style 3.1 ▪ 1 mark

The **moment** of a force is measured in:

A N

B N m

C N m^{-1}

D J

Solution 3.1

A N

B N m

C N m^{-1}

D J

Worked steps

B — N m.

Exam-style 3.2 ▪ 2 marks

A force of 50 N is applied at the end of a 0.40 m spanner, at $30°$ to the spanner. Calculate the **moment** about the nut.

Solution 3.2

Method: Centre of gravity and moment

Worked steps

$M = Fr\sin\theta = 50 \times 0.40 \times \sin 30^\circ = 50 \times 0.40 \times 0.50 = 10$ N m M1 A1.

Exam-style 3.3 ▪ 2 marks

Two parallel forces, each 8.0 N, act in opposite directions 0.25 m apart, forming a couple. Calculate the **torque**.

Solution 3.3

Method: Couple and torque

Worked steps

$\tau = Fd = 8.0 \times 0.25 = 2.0$ N m M1 A1.

Exam-style 3.4 ▪ 2 marks

Explain why two **equal** forces acting in the **same** direction do **not** form a couple.

Solution 3.4

Method: Couple and torque

Worked steps

Two equal forces in the **same** direction have a **resultant force** ($2F$, not zero) B1, so they cause straight-line acceleration (translation), not turning only — a couple needs them **opposite** B1.

Exam-style 3.5 ▪ 2 marks

A square aluminium sheet of **non-uniform** density has a mass of 1.6 kg and a uniform thickness of 3.0 mm. A thin rod of negligible mass is fixed through the centre of the sheet, and the sheet hangs vertically from the rod. To hold the sheet still, with one corner directly above the rod, a torque of 1.9 N m must be applied. In that position the centre of gravity of the sheet lies to the **right** of the rod.

(a) **Define** the **torque of a couple**.

(b) Explain what is meant by the **centre of gravity** of the sheet.

(c) **Show that** the centre of gravity of the sheet is a horizontal distance of about 0.12 m from the rod.

(d) **State and explain** whether the torque applied to the rod is clockwise or anticlockwise.

(e) The density of the aluminium is 2700 kg m^{-3}. **Show that** the side length of the square sheet is about 0.44 m.

Exam-style 3.5 ▪ 2 marks

(f) The rod is now removed and the sheet is hung freely from one corner. **State** where its centre of gravity lies relative to that corner.

(g) A second, uniform sheet of the same aluminium is 0.44 m square and 5.0 mm thick. **Determine** its weight.

Solution 3.5

Method: Finding a centre of gravity

Worked steps

(a) The torque of a couple is the product of **one of the forces** and the **perpendicular distance between the two lines of action** M1 A1.
(b) The single point at which the **whole weight** of the sheet may be taken to act B1.
(c) The applied torque balances the moment of the weight about the rod: $\tau = mgd$ M1, so $d = \dfrac{1.9}{1.6 \times 9.81} = 0.12$ m A1.
(d) **Anticlockwise** B1. The centre of gravity is to the right of the rod, so the weight acting downwards there turns the sheet **clockwise** about the rod; the applied torque must oppose it B1.

Solution 3.5

(e) $\rho = \dfrac{m}{V}$, so $V = \dfrac{1.6}{2700} = 5.93 \times 10^{-4}$ m^3 **M1**; for a square sheet $V = L^2 t$, so
$L^2 = \dfrac{5.93 \times 10^{-4}}{3.0 \times 10^{-3}} = 0.198$ m^2 **M1**; $L = 0.44$ m **A1**.
(f) Directly **below** that corner, on the vertical line through it **B1**.
(g) $V = L^2 t = (0.44)^2 \times 5.0 \times 10^{-3} = 9.68 \times 10^{-4}$ m^3 **M1**;
$m = \rho V = 2700 \times 9.68 \times 10^{-4} = 2.61$ kg **M1**; $W = mg = 2.61 \times 9.81 = 26$ N **A1**.