Exercise walk-through

Linear momentum and its conservation ▪ 3.3

eXercise

You must be able to

- state and apply **conservation of momentum** (using signed velocities)
- distinguish **elastic** and **inelastic** collisions
- use the elastic test: relative speed of **approach** = relative speed of **separation**
- **calculate** the percentage of kinetic energy transferred away in a collision
- apply momentum conservation to a **nuclear decay**, and say what else is conserved

Method — The principle

With **no resultant external force**, total momentum is **constant**. Always write momenta with **signs** (positive in one direction).
Explosion/recoil. A 2.0 kg and a 3.0 kg trolley spring apart from rest. Total momentum stays zero:

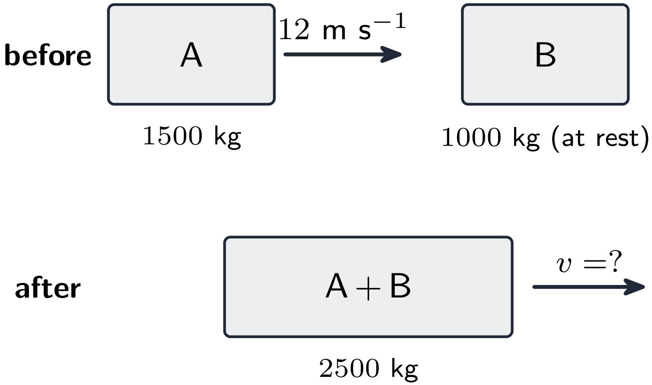
$$0 = (2.0)(6.0) + (3.0)(-v) \Rightarrow v = 4.0 \text{ m s}^{-1} \text{ (opposite way).}$$

Method — Inelastic collision (objects stick)

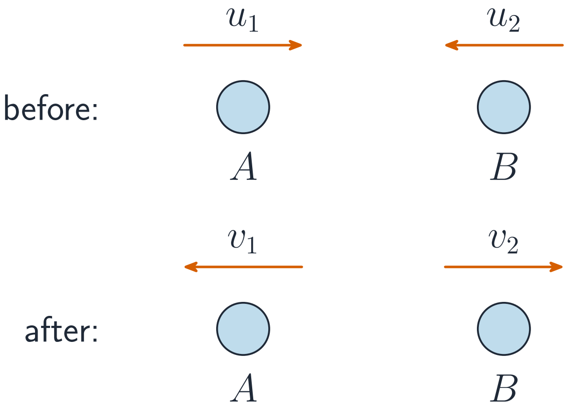
Momentum conserved, $v_1 = v_2 = v$:

$$1500(12) + 1000(0) = (2500)\,v \Rightarrow v = \frac{18\,000}{2500} = 7.2 \text{ m s}^{-1}.$$

Method — Inelastic collision (objects stick)



Method — Inelastic collision (objects stick)



Momentum is conserved in a head-on collision

Method — Elastic vs inelastic

- **Every** collision conserves **momentum**.
- An **elastic** collision **also** conserves **kinetic energy**; test: $u_1 - u_2 = -(v_1 - v_2)$ (relative speed of approach = of separation).
- An **inelastic** collision loses KE (to heat/sound/deformation); if they stick, it is perfectly inelastic.

Method — How much kinetic energy was lost?

"Elastic or not" is usually asked as a **number**. Work out the kinetic energy on each side and compare:

$$\%\ \text{transferred away} = \frac{E_{\text{K,before}} - E_{\text{K,after}}}{E_{\text{K,before}}} \times 100$$

Worked example (the two cars above). Before: $\frac{1}{2}(1500)(12)^2 = 1.08 \times 10^5$ J. After: $\frac{1}{2}(2500)(7.2)^2 = 6.48 \times 10^4$ J. So $\dfrac{1.08 \times 10^5 - 6.48 \times 10^4}{1.08 \times 10^5} \times 100 = 40\%$ is transferred to other forms. **Momentum is unchanged** all the same — that is the point of the question.

Method — A decay is an explosion

A stationary nucleus that decays is a two-body explosion, so the total momentum stays **zero**:

$$m_1 v_1 = m_2 v_2$$

The masses can be left in **atomic mass units** — they appear on both sides, so they cancel out of the ratio. Momentum, **charge (proton number)**, **nucleon number** and **total energy** are all conserved; kinetic energy is **not**, because energy is released from the nucleus.

Practice 2.1 ▪ 1 mark

State the **principle of conservation of momentum**.

Solution 2.1

Method: The principle

Worked steps

With **no resultant external force**, the **total momentum** of a system stays **constant** B1.

Practice 2.2 ▪ 2 marks

State what is conserved in (a) an **elastic** and (b) an **inelastic** collision.

Solution 2.2

Worked steps

(a) Elastic — **momentum and kinetic energy**; (b) inelastic — **momentum only** (KE decreases) **B1** **B1**.

Practice 2.3 ▪ 2 marks

Two trolleys, 2.0 kg and 3.0 kg, are pushed apart from rest by a spring. The 2.0 kg trolley moves off at 6.0 m s^{-1}. Find the speed of the 3.0 kg trolley.

Solution 2.3

Method: The principle

Worked steps

$0 = (2.0)(6.0) + (3.0)(-v)$ **M1** $\Rightarrow v = 12/3.0 = 4.0\ \text{m s}^{-1}$ **A1**.

Practice 2.4 ▪ 1 mark

State the test (using relative speeds) for an **elastic** collision.

Solution 2.4

Method: Elastic vs inelastic

Worked steps

The **relative speed of approach** equals the **relative speed of separation** B1.

Exam-style 3.1 ▪ 1 mark

In an **inelastic** collision, which quantity is conserved?

- **A** kinetic energy only
- **B** momentum only
- **C** both momentum and kinetic energy
- **D** neither

Solution 3.1

Method: Inelastic collision (objects stick)

A kinetic energy only

B momentum only ✓

C both momentum and kinetic energy

D neither

Worked steps

B — momentum only.

Exam-style 3.2 ▪ 3 marks

A 1500 kg car moving at 12 m s^{-1} hits a stationary 1000 kg car and they lock together. Calculate their **common velocity** just after the collision.

Solution 3.2

Method: Inelastic collision (objects stick)

Worked steps

$1500(12) + 1000(0) = (1500 + 1000)v$ **M1**; $18\,000 = 2500v$ **M1**; $v = 7.2$ m s^{-1} **A1**.

Exam-style 3.3 ▪ 2 marks

A 0.50 kg ball moving at 4.0 m s^{-1} collides **elastically** and head-on with a stationary 0.50 kg ball. State the velocity of **each** ball after the collision.

Solution 3.3

Method: Inelastic collision (objects stick)

Worked steps

Equal masses, elastic, target at rest: the first ball **stops** (0 m s^{-1}) and the second moves off at 4.0 m s^{-1} B1 B1.

Exam-style 3.4 ▪ 2 marks

Explain why the **kinetic energy** can decrease in a collision while the **momentum** does not.

Solution 3.4

Method: Elastic vs inelastic

Worked steps

In a collision the equal-and-opposite forces conserve total momentum **B1**; but kinetic energy can be transferred to **thermal/sound/deformation** energy, so total KE can decrease **B1**.

Exam-style 3.5 ▪ 2 marks

On a horizontal frictionless track, trolley A of mass 1.6 kg moves to the right at 4.5 m s^{-1}. Ahead of it, trolley B of mass 0.90 kg moves to the right at 1.5 m s^{-1}. A catches B, and the two move off together.

(a) **State** the principle of conservation of momentum.

(b) **Determine** the common velocity of the trolleys after the collision.

(c) **Calculate** the percentage of the total kinetic energy that is transferred to other forms during the collision.

(d) **State and justify** whether this collision is elastic.

Solution 3.5

Method: How much kinetic energy was lost?

Worked steps

(a) For a system with **no resultant external force** (an isolated system) B1, the **total momentum stays constant** — total momentum before = total momentum after B1.

(b) $p_{\text{before}} = (1.6)(4.5) + (0.90)(1.5) = 7.2 + 1.35 = 8.55$ kg m s^{-1} M1; after the collision the mass is $1.6 + 0.90 = 2.5$ kg M1; $v = \dfrac{8.55}{2.5} = 3.4$ m s^{-1} to the right A1.

(c) $E_{\text{K,before}} = \frac{1}{2}(1.6)(4.5)^2 + \frac{1}{2}(0.90)(1.5)^2 = 16.2 + 1.01 = 17.2$ J M1;
$E_{\text{K,after}} = \frac{1}{2}(2.5)(3.42)^2 = 14.6$ J M1; $\dfrac{17.2 - 14.6}{17.2} \times 100 = 15\%$ A1.

Solution 3.5

(d) **Not elastic** B1 — kinetic energy is not conserved: about 15% of it is transferred to thermal and sound energy and to deforming the trolleys B1.

Exam-style 3.6 ▪ 1 mark

A radon-220 nucleus, at rest and far from anything else, breaks apart into a polonium nucleus and an α-particle. Measurements show the polonium nucleus moves off at 2.2×10^5 m s^{-1}.

(a) **Complete** the equation for the decay.

$$^{220}_{86}\text{Rn} \rightarrow \,^{\underline{}}_{\underline{}}\text{Po} + \,^{4}_{2}\alpha$$

(b) By considering momentum, **calculate** the speed of the α-particle.

(c) **State** three quantities that are conserved as the nucleus breaks apart.

(d) **Determine** the ratio $\dfrac{\text{kinetic energy of the } \alpha\text{-particle}}{\text{kinetic energy of the polonium nucleus}}$, and comment on which fragment carries away most of the released energy.

Solution 3.6

Method: A decay is an explosion

Worked steps

(a) ${}^{220}_{86}\text{Rn} \rightarrow {}^{216}_{84}\text{Po} + {}^{4}_{2}\alpha$ — nucleon number $220 = 216 + 4$ and proton number $86 = 84 + 2$ **B1**.

(b) The nucleus is stationary, so the total momentum stays **zero** and the two fragments carry equal and opposite momentum **M1**: $4u \times v_\alpha = 216u \times 2.2 \times 10^5$ **M1**, so $v_\alpha = \dfrac{216 \times 2.2 \times 10^5}{4} = 1.2 \times 10^7$ m s^{-1}, in the opposite direction **A1**.
(The mass unit u cancels.)

(c) Any three of: **momentum**, **total energy**, **charge (proton number)**, **nucleon number** **B1** **B1** **B1**. Kinetic energy is **not** conserved — it is created from the

Solution 3.6

nucleus's store of energy.

(d) The momenta are equal in magnitude, and $E_K = \frac{p^2}{2m}$ M1, so for the same p the kinetic energy is **inversely proportional to the mass**: $\frac{E_\alpha}{E_{Po}} = \frac{m_{Po}}{m_\alpha} = \frac{216}{4} = 54$ M1 A1. The light α-particle carries away about 98% of the released energy — which is why it, not the recoiling nucleus, is what a detector sees.