Exercise walk-through

Non-uniform motion ▪ 3.2

eXercise

You must be able to

- describe **friction 摩擦力** and **drag 阻力** (drag grows with speed)
- explain the motion of an object falling through air
- explain **terminal velocity 收尾速度** and the energy transfer
- balance the **three** forces on an object falling through a **liquid** (weight, upthrust, drag)
- **draw** labelled force arrows, and get a terminal speed out of a given drag equation

Method — Drag increases with speed

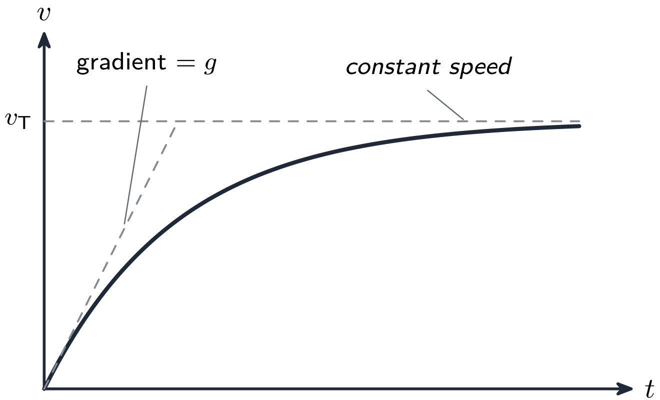
A **drag** (viscous) force from a fluid opposes motion. A simple model: **drag is zero at zero speed and grows as the speed grows**.

Method — Falling through air → terminal velocity

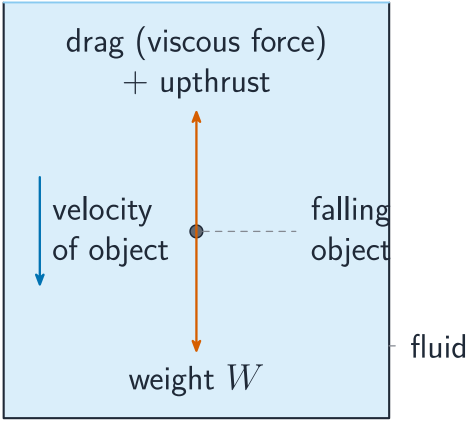
An object dropped from rest:

1. at first only **weight** acts → it accelerates at g;
2. as it speeds up, **drag grows**, so the resultant force and the acceleration **shrink**;
3. when **drag** = **weight**, the resultant force is zero → constant speed, the **terminal velocity**.

Method — Falling through air → terminal velocity



Method — Falling through air → terminal velocity



Forces on a falling object reaching terminal velocity

Method — Energy at terminal velocity

At terminal velocity the kinetic energy is **constant**, so the lost **gravitational potential energy** is transferred to **thermal energy of the air — not** to extra KE of the faller.

Method — Falling through a LIQUID: three forces, not two

In air the upthrust is tiny and we ignore it. In a **liquid** it is not, so **three** forces act on a sinking object:

- **weight** $W = mg$, down (or $\rho_{\text{object}} Vg$);
- **upthrust** U, up — the weight of the liquid pushed aside, $U = \rho_{\text{liquid}} Vg$;
- **viscous drag** D, up, growing with speed.

Upthrust exists because the **pressure in a liquid increases with depth**: the pressure on the bottom of the object is larger than on the top, so the upward force on the bottom beats the downward force on the top, and the difference is the upthrust.

At **terminal velocity** the object is not accelerating, so

$$W = U + D$$

Method — Falling through a LIQUID: three forces, not two

Worked example. A sphere of volume 2.0×10^{-7} m^3 and mass 1.6×10^{-3} kg falls through oil of density 900 kg m^{-3}. The drag is $D = kv$ with $k = 1.4 \times 10^{-2}$ kg s^{-1}.

$$W = mg = 1.6 \times 10^{-3} \times 9.81 = 1.57 \times 10^{-2} \text{ N}$$

$$U = \rho V g = 900 \times 2.0 \times 10^{-7} \times 9.81 = 1.77 \times 10^{-3} \text{ N}$$

$$D = W - U = 1.39 \times 10^{-2} \text{ N} \quad \Rightarrow \quad v = \frac{D}{k} = \frac{1.39 \times 10^{-2}}{1.4 \times 10^{-2}} = 0.99 \text{ m s}^{-1}$$

Write $W = U + D$ **first** — that line is the method mark, whatever happens to the arithmetic afterwards.

Method — Falling through a LIQUID: three forces, not two

A **heavier** (or denser) object reaches a **higher** terminal velocity — it needs more speed (more drag) to balance its larger weight. Opening a **parachute** suddenly increases the drag, so drag exceeds weight and the diver **decelerates** to a new, much **lower** terminal velocity.

Practice 2.1 ▪ 1 mark

State what happens to the **drag force** on an object as its speed increases.

Solution 2.1

Method: Falling through a LIQUID: three forces, not two

Worked steps

It **increases** as the speed increases (zero at zero speed) **B1**.

Practice 2.2 ▪ 1 mark

State the condition (in terms of forces) for an object to move at **terminal velocity**.

Solution 2.2

Worked steps

The **drag (resistive) force equals the weight**, so the **resultant force is zero** B1.

Practice 2.3 ▪ 1 mark

State the acceleration of an object **just after** it is released and begins to fall through air.

Solution 2.3

Method: Falling through air $\rightarrow$ terminal velocity

Worked steps

g (about 9.81 m s^{-2}) — the maximum acceleration, since drag is still zero **B1**.

Practice 2.4 ▪ 1 mark

State the main energy transfer for a parachutist falling at **terminal velocity**.

Solution 2.4

Worked steps

Gravitational potential energy $\rightarrow$ **thermal energy of the air** B1.

Exam-style 3.1 ▪ 1 mark

A parachutist falls at constant (terminal) velocity. Which statement is **not** correct?

A Gravitational PE is converted to kinetic energy of the air.

B Gravitational PE is converted to kinetic energy of the parachutist.

C Gravitational PE is converted to thermal energy of the air.

D The resultant force on the parachutist is zero.

Solution 3.1

A Gravitational PE is converted to kinetic energy of the air.

B Gravitational PE is converted to kinetic energy of the parachutist. ✓

C Gravitational PE is converted to thermal energy of the air.

D The resultant force on the parachutist is zero.

Worked steps

B — at terminal velocity the KE of the parachutist is constant, so PE is **not** converted to her KE.

Exam-style 3.2 ▪ 4 marks

Describe and explain the motion of an object dropped from rest and falling through air, until it reaches terminal velocity.

Solution 3.2

Method: Falling through air $\rightarrow$ terminal velocity

Worked steps

Just after release only weight acts, so acceleration $= g$ **B1**; as speed rises, drag increases **B1**, so the resultant force and acceleration **decrease** **B1**; when drag $=$ weight the resultant force is zero and the speed is constant — terminal velocity **B1**.

Exam-style 3.3 ▪ 2 marks

A skydiver is falling at terminal velocity. State the **resultant force** on her and explain why her velocity stays **constant**.

Solution 3.3

Method: Falling through air $\rightarrow$ terminal velocity

Worked steps

The resultant force is **zero** B1; with no resultant force there is no acceleration, so the velocity stays constant (Newton's first law) B1.

Exam-style 3.4 ▪ 2 marks

Sketch a **velocity–time graph** for an object falling from rest through air, labelling the terminal velocity.

Solution 3.4

Method: Falling through air $\longrightarrow$ terminal velocity

Worked steps

Curve from the origin rising with an initial gradient (steepest at the start) **B1**, bending and **flattening** to a horizontal line at the terminal velocity (labelled) **B1**.

Exam-style 3.5 ▪ 1 mark

A skydiver falling at terminal velocity opens her parachute.

(a) State what happens to her velocity **just after** the parachute opens.

(b) Explain, in terms of **forces**, why she settles to a new **lower** terminal velocity.

Solution 3.5

Method: Falling through a LIQUID: three forces, not two

Worked steps

(a) Her velocity **decreases** (she decelerates) B1. (b) The parachute greatly increases the **drag**, so drag now exceeds weight and the resultant force is **upward** B1; as she slows, the drag falls until **drag = weight** again, giving a new, lower terminal velocity B1.

Exam-style 3.6 ▪ 1 mark

A small steel sphere of radius 3.0 mm is released from rest just below the surface of a tall jar of oil. The density of steel is 7800 kg m^{-3} and the density of the oil is 900 kg m^{-3}.

(a) **Define** pressure.

(b) **Explain** how the pressure in the oil gives rise to an **upthrust** on the sphere.

(c) On the diagram above, **draw** labelled arrows from the sphere to show the **three** forces acting on it as it falls.

(d) The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$. **Show that** the upthrust on the sphere is about 1.0×10^{-3} N.

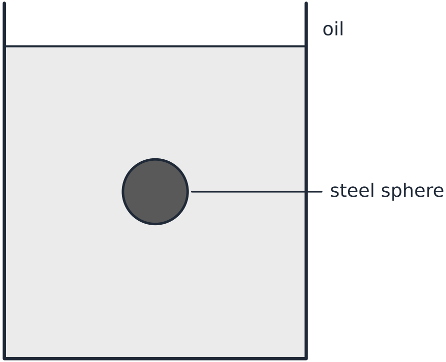
(e) The viscous drag on the sphere is given by $D = 6\pi\eta r v$, where v is its speed. **Determine** the SI base units of η.

(f) The oil has $\eta = 0.20$ kg m^{-1} s^{-1}. **Determine** the terminal speed of the sphere.

Exam-style 3.6 ▪ 1 mark

(g) **Calculate** the resultant force on the sphere at the instant it is released from rest, and state its direction.

Exam-style 3.6 ▪ 1 mark



Solution 3.6

Method: Falling through a LIQUID: three forces, not two

Worked steps

(a) Pressure is the **normal force per unit area** B1.
(b) The pressure in a liquid **increases with depth** B1, so the upward force on the bottom of the sphere is greater than the downward force on its top, and the difference is the upthrust B1.
(c) An arrow vertically **downwards** labelled *weight* B1; an arrow vertically **upwards** labelled *upthrust* B1; an arrow vertically **upwards** labelled *(viscous) drag* B1.
(d) $V = \frac{4}{3}\pi(3.0 \times 10^{-3})^3 = 1.13 \times 10^{-7}$ m^3 M1;
$U = \rho_{\text{oil}} Vg = 900 \times 1.13 \times 10^{-7} \times 9.81 = 1.0 \times 10^{-3}$ N A1.

Solution 3.6

(e) $\eta = \dfrac{D}{6\pi r v} = \dfrac{\text{kg m s}^{-2}}{\text{m} \times \text{m s}^{-1}}$ **M1** $= \text{kg m}^{-1}\,\text{s}^{-1}$ **A1**. (6π has no units.)
(f) Weight $= \rho_{\text{steel}} Vg = 7800 \times 1.13 \times 10^{-7} \times 9.81 = 8.7 \times 10^{-3}$ N **M1**; at terminal velocity $W = U + D$, so $D = 8.7 \times 10^{-3} - 1.0 \times 10^{-3} = 7.7 \times 10^{-3}$ N **M1**;
$v = \dfrac{D}{6\pi\eta r} = \dfrac{7.7 \times 10^{-3}}{6\pi(0.20)(3.0 \times 10^{-3})} = 0.68$ m s^{-1} **A1**.
(g) At rest the speed is zero, so the **drag is zero** and only weight and upthrust act: resultant $= 8.7 \times 10^{-3} - 1.0 \times 10^{-3} = 7.7 \times 10^{-3}$ N **M1**, directed **downwards** **A1**. (This is the same size as the drag at terminal velocity, which is why the sphere ends up moving at 0.68 m s^{-1}.)