Exercise walk-through

Production and use of ultrasound ▪ 24.1

eXercise

You must be able to

- explain the **piezo-electric** transducer (send and detect)
- use **acoustic impedance** $Z = \rho c$ and the **reflection coefficient**
- use the attenuation $I = I_0 e^{-\mu x}$

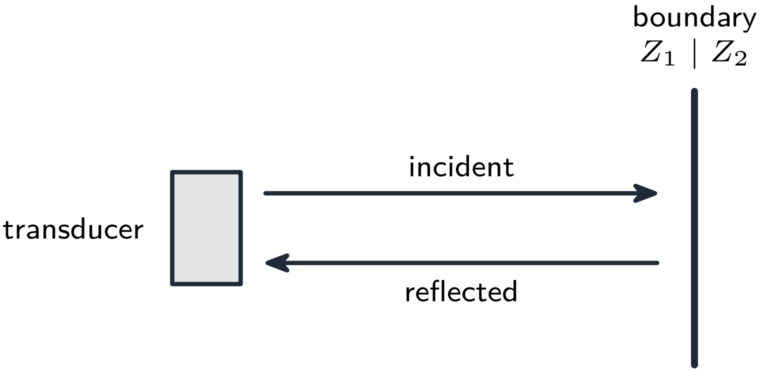
Method — Transducer and pulse-echo

A **piezo-electric** crystal changes shape when a p.d. is applied (to **emit** ultrasound) and produces an e.m.f. when squeezed (to **detect** it) — so one transducer both sends and receives. A pulse partly **reflects** at a boundary between tissues:

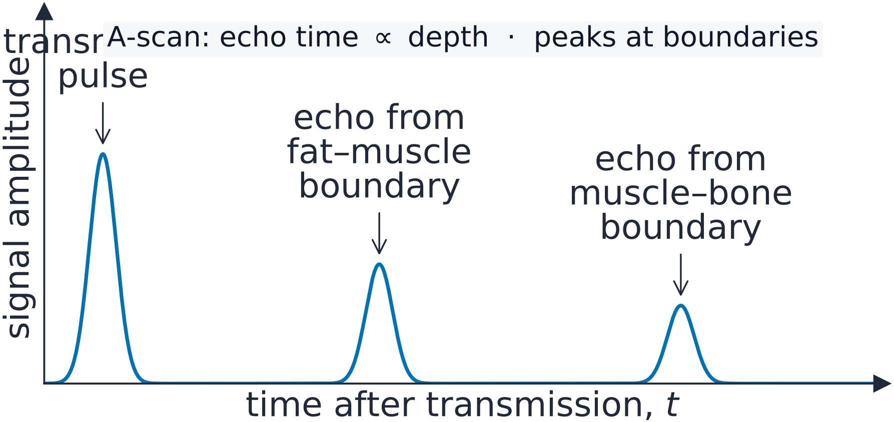
$$Z = \rho c, \qquad \frac{I_{\mathrm{R}}}{I_0} = \frac{(Z_1 - Z_2)^2}{(Z_1 + Z_2)^2}, \qquad I = I_0 e^{-\mu x}.$$

A large impedance **mismatch** reflects strongly — which is why a **coupling gel** removes the air gap (air's Z is very different).

Method — Transducer and pulse-echo



Method — Transducer and pulse-echo



An ultrasound A-scan trace

Practice 2.1 ▪ 2 marks

Describe the **piezo-electric effect**.

Solution 2.1

Method: Transducer and pulse-echo

Worked steps

Applying a p.d. across the crystal makes it **change shape** B1; changing its shape makes it produce an **e.m.f.** B1.

Practice 2.2 ▪ 1 mark

Define the **specific acoustic impedance** of a medium.

Solution 2.2

Method: Transducer and pulse-echo

Worked steps

$Z = \rho c$, the product of the medium's density and the speed of sound in it **B1**.

Practice 2.3 ▪ 2 marks

Explain how **one** transducer can both send and detect ultrasound.

Solution 2.3

Method: Transducer and pulse-echo

Worked steps

An alternating p.d. makes the crystal **vibrate**, emitting ultrasound B1; a returning pulse makes it vibrate and produce an **e.m.f.**, which is detected B1.

Practice 2.4 ▪ 1 mark

Write the equation for the **attenuation** of ultrasound.

Solution 2.4

Worked steps

$I = I_0 e^{-\mu x}$ **B1**.

Exam-style 3.1 ▪ 1 mark

For a **strong** reflection at a tissue boundary, the two acoustic impedances must be:

- **A** equal
- **B** very different
- **C** both zero
- **D** both very large

Solution 3.1

Method: Transducer and pulse-echo

A equal

B very different ✓

C both zero

D both very large

Worked steps

B — very different.

Exam-style 3.2 ▪ 3 marks

Calculate the intensity reflection coefficient for a boundary where $Z_1 = 1.6 \times 10^6$ and $Z_2 = 1.7 \times 10^6$ (SI units).

Solution 3.2

Worked steps

$$\frac{I_{\mathrm{R}}}{I_0} = \frac{(1.6 - 1.7)^2}{(1.6 + 1.7)^2} = \frac{(0.1)^2}{(3.3)^2} \quad \boxed{\textbf{M1}}\boxed{\textbf{M1}} = 9.2 \times 10^{-4} \quad \boxed{\textbf{A1}}.$$

Exam-style 3.3 ▪ 2 marks

Explain why a **coupling gel** is used between the transducer and the skin.

Solution 3.3

Method: Transducer and pulse-echo

Worked steps

Without gel there is an **air gap**, whose acoustic impedance is very different from skin B1, so almost all the ultrasound would be reflected; the gel removes the air gap so the ultrasound enters the body B1.

Exam-style 3.4 ▪ 2 marks

Ultrasound of intensity I_0 passes through 5.0 cm of tissue with $\mu = 23$ m^{-1}. Calculate I/I_0.

Solution 3.4

Method: Transducer and pulse-echo

Worked steps

$I/I_0 = e^{-\mu x} = e^{-(23)(0.050)} = e^{-1.15}$ **M1** $= 0.32$ **A1**.

Exam-style 3.5 ▪ 1 mark

An ultrasound scanner sends a pulse into a patient. The specific acoustic impedance of soft tissue is 1.63×10^6 kg m^{-2} s^{-1}, of bone 6.40×10^6 kg m^{-2} s^{-1} and of air 4.29×10^2 kg m^{-2} s^{-1}.

(a) **Define specific acoustic impedance**.

(b) **Explain** how a piezoelectric crystal **detects** the reflected ultrasound.

(c) **Determine** the intensity reflection coefficient at a soft tissue-bone boundary.

(d) **Determine** the intensity reflection coefficient at a soft tissue-air boundary, and **explain** why a gel is smeared on the skin before a scan.

(e) The pulse returns from a boundary 9.6 cm deep after 1.2×10^{-4} s. **Determine** the speed of ultrasound in the tissue.

Solution 3.5

Method: Transducer and pulse-echo

Worked steps

(a) The product of the **density** of the medium and the **speed of sound** in it, $Z = \rho c$ B1.

(b) The reflected wave's pressure variation makes the crystal **change shape** B1, and a piezoelectric crystal produces a **potential difference** across its faces when it is deformed, giving an electrical signal that varies with the received wave B1.

(c) $\dfrac{I_R}{I_0} = \dfrac{(Z_2 - Z_1)^2}{(Z_2 + Z_1)^2}$ M1 $= \dfrac{(6.40 - 1.63)^2}{(6.40 + 1.63)^2}$ M1 $= \dfrac{22.8}{64.5} = 0.35$ A1.

(d) With air, $\dfrac{(1.63 \times 10^6 - 429)^2}{(1.63 \times 10^6 + 429)^2} \approx 0.999$ M1 A1 — essentially **all** the ultrasound is

Solution 3.5

reflected at a skin-air boundary. Gel excludes the air layer and has an impedance close to that of skin, so the pulse actually enters the body **B1**.

(e) The pulse travels **there and back**, a distance of $2 \times 0.096 = 0.192$ m **M1**;

$c = \dfrac{0.192}{1.2 \times 10^{-4}} = 1.6 \times 10^{3}$ m s^{-1} **A1**.