Exercise walk-through

Mass defect and nuclear binding energy ▪ 23.1

eXercise

You must be able to

- use $E = mc^2$ and balance **nuclear equations**
- define **mass defect** and **binding energy**
- use the **binding energy per nucleon** curve for fusion and fission

Method — Mass-energy and binding energy

$$E = mc^2, \qquad \Delta E = c^2 \Delta m, \qquad 1\ \text{u} \approx 931\ \text{MeV}.$$

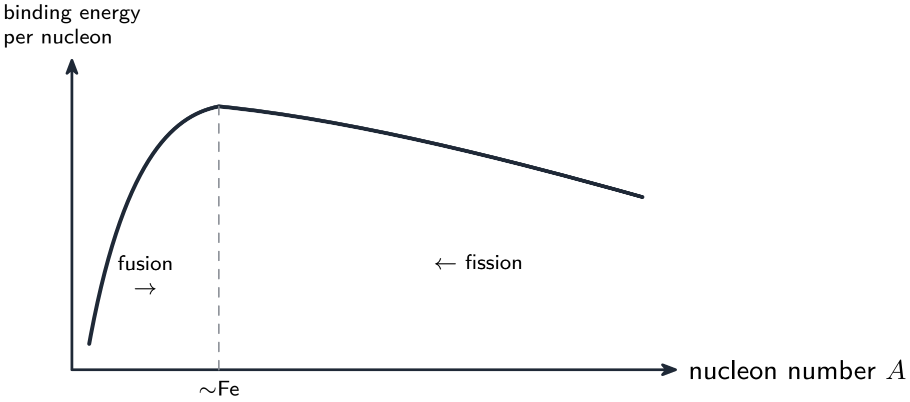
The **mass defect** is the difference between the mass of the separate nucleons and the mass of the nucleus; the **binding energy** is the energy equivalent of this ($\Delta E = c^2 \Delta m$) — the energy to split the nucleus apart.

Method — Binding energy per nucleon

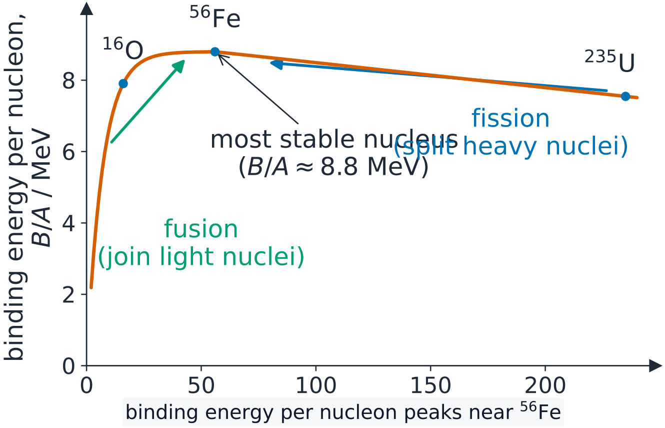
The peak (near iron) is the **most stable**. **Fusion** of light nuclei and **fission** of heavy nuclei both move **towards the peak** — increasing binding energy per nucleon and **releasing energy**.

In nuclear equations, **nucleon number** and **charge** are conserved.

Method — Binding energy per nucleon



Method — Binding energy per nucleon



Binding energy per nucleon across the nuclides

Method — Energy released in a decay (from masses)

The energy released is the **mass lost** (parent mass − total product mass), converted with 1 u ≈ 931 MeV:

$$Q = \left(m_{\text{before}} - m_{\text{after}}\right) \times 931\ \text{MeV/u}.$$

Worked example. ${}^{238}_{92}\text{U} \to {}^{234}_{90}\text{Th} + {}^{4}_{2}\alpha$, with masses (u): U $= 238.05079$, Th $= 234.04360$, $\alpha = 4.00260$. Mass lost $\Delta m = 238.05079 - (234.04360 + 4.00260) = 0.00459$ u; energy released $Q = 0.00459 \times 931 = 4.3$ MeV.

Working **backwards**: if the released energy is known, $\Delta m = Q/931$ u, which fixes a missing mass.

Practice 2.1 ▪ 1 mark

State **Einstein's** mass–energy equation.

Solution 2.1

Worked steps

$E = mc^2$ **B1**.

Practice 2.2 ▪ 1 mark

Define **mass defect**.

Solution 2.2

Method: Mass-energy and binding energy

Worked steps

The difference between the **total mass of the separate nucleons** and the **mass of the nucleus** B1.

Practice 2.3 ▪ 1 mark

Define **binding energy**.

Solution 2.3

Worked steps

The energy needed to **separate a nucleus** into its individual nucleons (equivalent to the mass defect) **B1**.

Practice 2.4 ▪ 1 mark

In ${}^{14}_{7}\mathrm{N} + {}^{4}_{2}\mathrm{He} \rightarrow {}^{A}_{8}\mathrm{O} + {}^{1}_{1}\mathrm{H}$, find the nucleon number A.

Solution 2.4

Worked steps

$14 + 4 = A + 1 \Rightarrow A = 17$ B1.

Exam-style 3.1 ▪ 1 mark

The **most stable** nuclei have:

A the highest mass

B the highest binding energy per nucleon

C the lowest binding energy per nucleon

D zero binding energy

Solution 3.1

Method: Binding energy per nucleon

A the highest mass

B the highest binding energy per nucleon ✓

C the lowest binding energy per nucleon

D zero binding energy

Worked steps

B — the highest binding energy per nucleon.

Exam-style 3.2 ▪ 2 marks

A reaction has a mass defect of 0.030 u. Calculate the energy released.
(1 u $= 931$ MeV.)

Solution 3.2

Worked steps

$E = 0.030 \times 931 = 28$ MeV M1 A1.

Exam-style 3.3 ▪ 3 marks

Explain how the binding-energy-per-nucleon curve shows that **both** fusion of light nuclei **and** fission of heavy nuclei can release energy.

Solution 3.3

Method: Binding energy per nucleon

Worked steps

Both light nuclei (fusing) and heavy nuclei (splitting) move **towards the peak** of the curve B1; this **increases** the binding energy per nucleon B1; the products are more tightly bound, so energy is **released** B1.

Exam-style 3.4 ▪ 2 marks

Calculate the energy equivalent of a mass change of 1.0×10^{-29} kg.

Solution 3.4

Worked steps

$E = mc^2 = (1.0 \times 10^{-29})(3.0 \times 10^8)^2$ **M1** $= 9.0 \times 10^{-13}$ J **A1**.

Exam-style 3.5 ▪ 3 marks

In the decay $^{210}_{84}\text{Po} \rightarrow\ ^{206}_{82}\text{Pb} +\ ^{4}_{2}\alpha$, the masses (u) are: Po $= 209.98286$, Pb $= 205.97444$, $\alpha = 4.00260$. Calculate the energy released, in MeV. (1 u $= 931$ MeV.)

Solution 3.5

Method: Energy released in a decay (from masses)

Worked steps

Mass lost $\Delta m = 209.98286 - (205.97444 + 4.00260) = 0.00582$ u **M1** **A1**; energy released $Q = 0.00582 \times 931 = 5.4$ MeV **A1**.

Exam-style 3.6 ▪ 1 mark

Deuterium nuclei fuse to form helium-3 in the reaction

$$^{2}_{1}\mathrm{H} + ^{2}_{1}\mathrm{H} \rightarrow\ ^{3}_{2}\mathrm{He} + ^{1}_{0}\mathrm{n}$$

The masses are: deuterium 2.01410 u, helium-3 3.01603 u, neutron 1.00867 u. Take 1 u $= 1.66 \times 10^{-27}$ kg, $c = 3.00 \times 10^{8}$ m s^{-1} and 1 MeV $= 1.60 \times 10^{-13}$ J.

(a) **State** what is meant by the **mass defect** of a nucleus.

(b) **Determine** the mass difference of this reaction, in u.

(c) **Determine** the energy released, in joules and in MeV.

(d) **Determine** the energy released, in MeV, when 1.0 g of deuterium is fused this way.

(e) **Explain** why fusion releases energy for light nuclei but fission releases energy for heavy ones, referring to **binding energy per nucleon**.

Solution 3.6

Method: Binding energy per nucleon

Worked steps

(a) The **difference** between the total mass of the separate nucleons and the mass of the assembled nucleus **B1**.
(b) Left: $2 \times 2.01410 = 4.02820$ u; right: $3.01603 + 1.00867 = 4.02470$ u **M1**; difference $= 0.00350$ u **A1**.
(c) $\Delta m = 0.00350 \times 1.66 \times 10^{-27} = 5.81 \times 10^{-30}$ kg **M1**;
$E = \Delta mc^2 = 5.81 \times 10^{-30} \times (3.00 \times 10^8)^2 = 5.2 \times 10^{-13}$ J **A1**; dividing by 1.60×10^{-13} gives 3.3 MeV **A1**.
(d) 1.0 g of deuterium contains $\dfrac{1.0 \times 10^{-3}}{2.01410 \times 1.66 \times 10^{-27}} = 3.0 \times 10^{23}$ nuclei **M1**; the

Solution 3.6

reaction consumes **two** nuclei per event, so there are 1.5×10^{23} events M1; total $= 1.5 \times 10^{23} \times 3.3 = 4.9 \times 10^{23}$ MeV A1.

(e) Binding energy per nucleon rises steeply from hydrogen to a maximum near iron, then falls slowly for heavier nuclei B1. Energy is released whenever the products have a **greater** binding energy per nucleon than the reactants B1. So joining light nuclei moves up the curve and releases energy, while splitting a heavy nucleus moves up it from the other side and does the same B1.