Past-paper walk-through

Characteristics of alternating currents ▪ 21.1

eXercise

Questions in this set

- 9702/42 N25 Q7 ▪ 8 marks
- 9702/41 J24 Q7 ▪ 10 marks
- 9702/42 N23 Q4 ▪ 11 marks
- 9702/43 N23 Q7 ▪ 8 marks

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = “or words to that effect” – the wording need not match.

Question 7

9702/42 N25 ▪ Q7 ▪ 8 marks

An alternating voltage V varies with time t according to

$$V = 18\cos 40\pi t$$

where V is in V and t is in s.

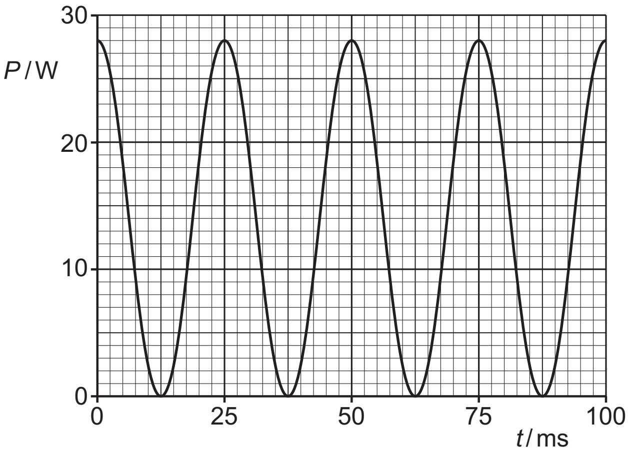
(a)(i) show that the period is 0.050 s **[1]**

(a)(ii) determine the root-mean-square (r.m.s.) voltage. **[1]**

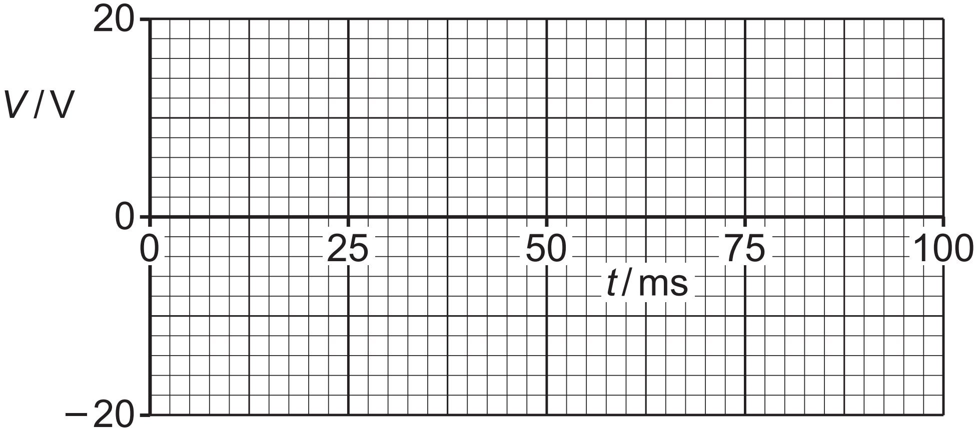
(b) On Fig. 7.1, sketch the variation of V with t for values of t from $t = 0$ to $t = 100$ ms. **[3]**

(c) The alternating voltage is rectified to produce an output voltage across a load resistor R, as shown in Fig. 7.2. **[3]**

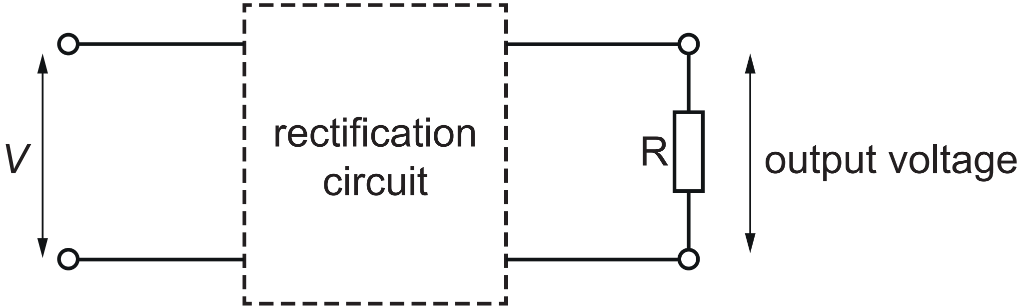
Question 7



Question 7



Question 7



Solution 7

(a)(i)

Worked steps

- $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{40\pi}$
- $T = 0.050$ s, as required.

Mark scheme

- T $= 2\pi$ / 40π $=$ 0.050 s **A1**

Solution 7

(a)(ii) Worked steps

- $V_{\mathsf{rms}} = \dfrac{V_0}{\sqrt{2}} = \dfrac{18}{\sqrt{2}}$
- $V_{\mathsf{rms}} = 13\ \mathsf{V}$

Solution 7

Mark scheme

- $V_{r.m.s.} = 18 / \sqrt{2}$
- $= 13$ V **A1**

Solution 7

(b) Worked steps

Three separate marks: the period, the phase and the amplitude — check each before moving on.

- Draw a sinusoid of period 50 ms, so exactly **two** cycles fit between $t = 0$ and $t = 100$ ms.
- Get the phase right: this one starts at its maximum, so peaks fall at $t = 0$, 50 and 100 ms, and minima at 25 and 75 ms.
- Mark the maximum and minimum values as $+18$ V and -18 V — the peak, not the r.m.s. value you just calculated.

Solution 7

Mark scheme

- sinusoidal curve of period 50 ms from $t = 0$ to $t = 100$ ms **B1**
- correct phase (VMAX at $t = 0$, 50, 100 ms and –VMAX at 25, 75 ms etc.) **B1**
- maximum and minimum voltages shown as ± 18 V **B1**

Solution 7

(c) Mark scheme

- Any three points from:
- ▪ rectification is full-wave
- ▪ mean power = 14 W
- ▪ resistance of R = 12 Ω
- ▪ peak current in R = 1.6 A or r.m.s. current in R = 1.1 A
- ▪ period of output voltage / power = 25 ms or frequency of output voltage / power = 40 Hz or angular frequency of output voltage / power = 250 rad s–1 **B3**

Question 7

9702/41 J24 ▪ Q7 ▪ 10 marks

A circuit contains a power supply that provides a sinusoidal alternating input voltage V_{IN}. There is an output voltage V_{OUT} across a load resistor R, as shown in Fig. 7.1.

(a) State the purpose of the circuit in Fig. 7.1. **[2]**

(b)(i) The load resistor R has a resistance of $370\ \Omega$.

Show that the maximum power dissipated in R is 0.22 W. **[2]**

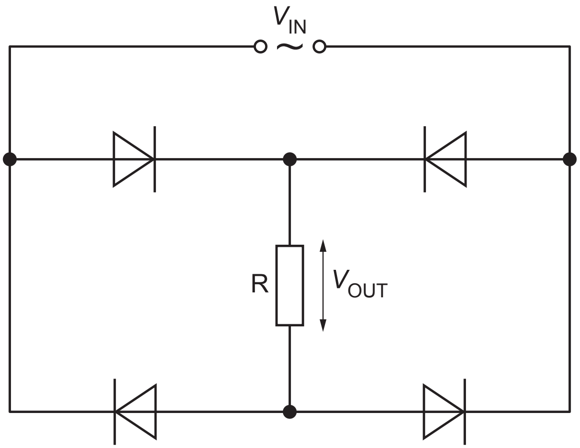
(b)(ii) On Fig. 7.3, sketch the variation with *t* of the power *P* dissipated in R. **[3]**

(b)(iii) Calculate the mean power dissipated in R. **[1]**

(c) The circuit of Fig. 7.1 is disconnected, and R is connected directly across the power supply.

Explain, without calculation, how the mean power now dissipated in R compares with the answer in (b)(iii). **[2]**

Question 7



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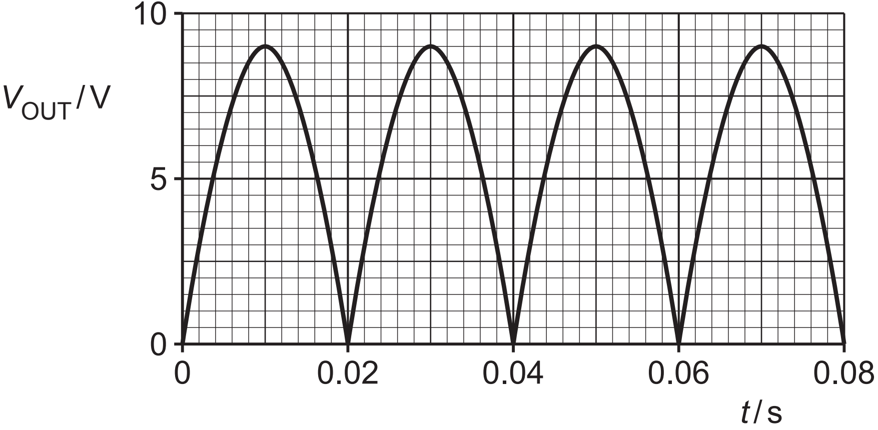

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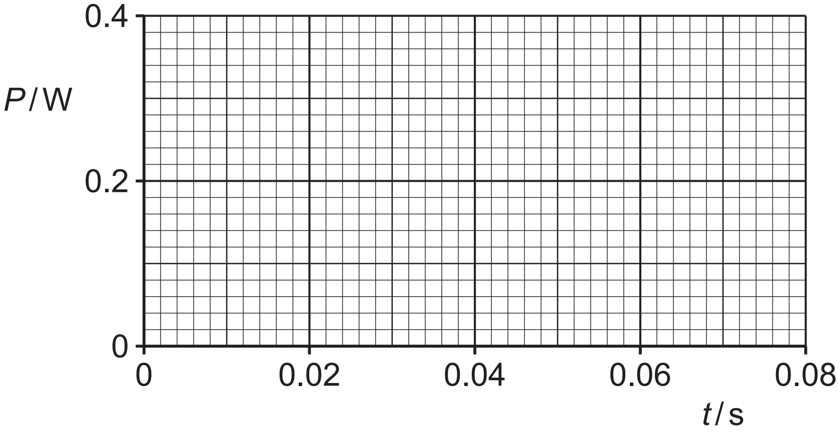


Solution 7

(a)

Mark scheme

- rectification (of the input voltage)
- full-wave

Solution 7

(b)(i) Worked steps

Know. "Maximum power" means the instant the rectified voltage across R is at its **peak**. Read that peak off the output graph: $V_0 = 9.0$ V.

Equation. $P = \dfrac{V^2}{R}$.

Substitute. $P_{\text{MAX}} = \dfrac{9.0^2}{370}$.

Answer. $P_{\text{MAX}} = 0.22$ W, as required.

Solution 7

Check. Use the **peak** voltage here, not the r.m.s. value. Dividing 9.0 by $\sqrt{2}$ first would give 0.11 W, which is the *mean* power asked for two parts later. Both numbers are correct physics; only one answers this question.

Mark scheme

- $P = V^2/R$
- or
- maximum $V = 9.0$ V
- $P_{\mathsf{MAX}} = 9.0^2/370 = 0.22$ W

Solution 7

(b)(ii) Worked steps

Know. Power is $\frac{V^2}{R}$, and squaring removes every minus sign. So the power curve never goes below zero: it just touches the time axis each time the voltage passes through zero.

Shape. A row of identical humps sitting on the axis. Each hump rises from zero, reaches 0.22 W, and falls back to zero.

Timing. The rectified voltage is zero at $t = 0,\ 0.02,\ 0.04,\ 0.06$ and 0.08 s, so put a minimum at each of those. The maxima fall halfway between, at $t = 0.01,\ 0.03,\ 0.05$ and 0.07 s, and every one of them is drawn at 0.22 W.

Solution 7

Check. The power repeats **twice as often** as the input voltage, because squaring turns each negative half-cycle into a second positive hump. A sketch with only half as many humps is the commonest error, and it loses the frequency mark.

Mark scheme

- sinusoidal shape with minima sitting on the time axis
- correct frequency and phase, with minima at 0, 0.02, 0.04, 0.06 and 0.08 s and maxima at 0.01, 0.03, 0.05 and 0.07 s
- all maxima shown at 0.22 W

Solution 7

(b)(iii) Worked steps

Know. For a sinusoidal quantity that has been squared, the mean sits exactly halfway between the peak and zero. So the mean power is **half** the peak power.

Equation. $\langle P \rangle = \dfrac{P_{\mathsf{MAX}}}{2}$.

Substitute. $\langle P \rangle = \dfrac{0.22}{2}$.

Answer. $\langle P \rangle = 0.11$ W.

Solution 7

Check. This is the same statement as $P = \dfrac{V_{\text{r.m.s.}}^2}{R}$ with $V_{\text{r.m.s.}} = \dfrac{V_0}{\sqrt{2}}$: squaring the $\sqrt{2}$ gives the factor of two. Either route earns the mark.

Mark scheme

- mean power = peak power / 2 = 0.22 / 2
- $= 0.11$ W

Solution 7

(c)

Mark scheme

- power–time graph is identical
- (so) mean powers are equal

Question 4

9702/42 N23 ▪ Q4 ▪ 11 marks

An electron in a metal rod moves randomly about a mean position. When an alternating voltage is applied to the ends of the rod, the mean position can be considered to oscillate with simple harmonic motion along the axis of the rod. Fig. 4.1 shows the variation with time *t* of the displacement *x* of the mean position from a fixed point on the axis of the rod.

(a)(i) Determine the amplitude of the oscillations. **[1]**

(a)(ii) Determine the angular frequency of the oscillations. **[1]**

(a)(iii) Use your answers in **(a)(i)** and **(a)(ii)** to show that the maximum drift speed v_0 of the electron is 1.1×10^{-7} m s^{-1}. **[2]**

(b)(i) Use the information in **(a)(iii)** to calculate the magnitude I_0 of the maximum current in the rod. **[2]**

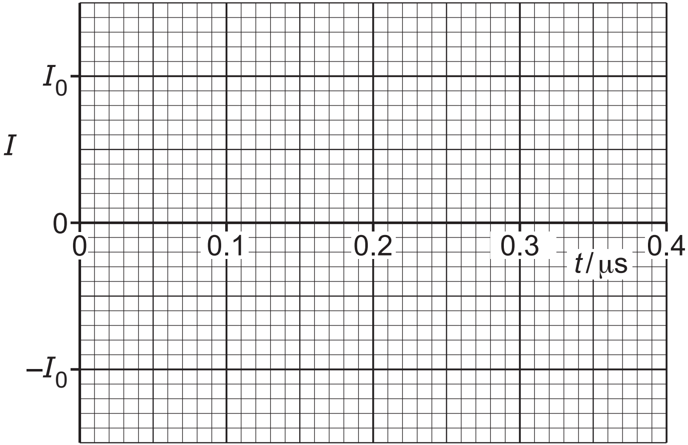
Question 4

(b)(ii) On Fig. 4.2, sketch the variation of the current I in the rod with time t between $t = 0$ and $t = 0.40\mu$s. **[2]**

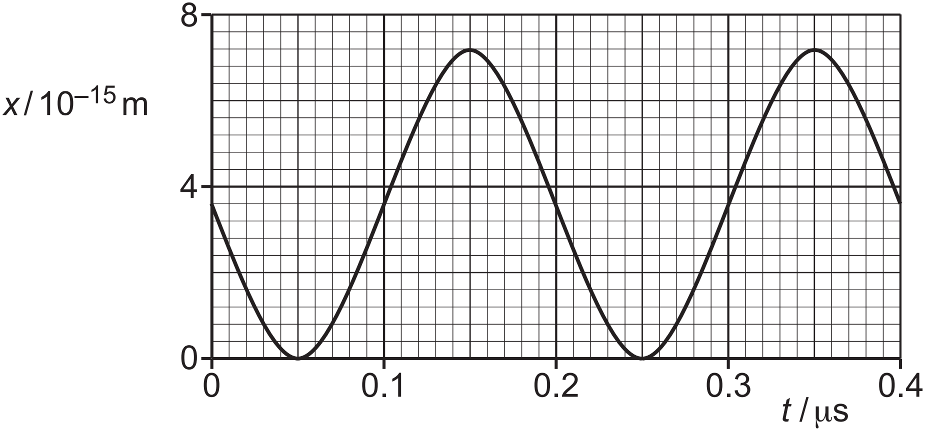
(b)(iii) Use your answers in **(a)(ii)** and **(b)(i)** to determine an expression for I in terms of t, where I is in A and t is in s. **[2]**

(b)(iv) Determine the root-mean-square (r.m.s.) current in the rod. **[1]**

Question 4



Question 4



Solution 4

(a)(i) Worked steps

Know. The graph shows the displacement swinging from one extreme to the other. That full swing is **twice** the amplitude, so halve it.
Read. Peak-to-peak displacement $= 7.2 \times 10^{-15}$ m.
Answer. $x_0 = \frac{1}{2} \times 7.2 \times 10^{-15} = 3.6 \times 10^{-15}$ m.

Solution 4

Check. Amplitude is measured from the **middle** of the motion to one extreme, never from one extreme to the other. Forgetting the half doubles every answer that follows.

Mark scheme

- amplitude $= ½ \times 7.2 \times 10–15$
- $= 3.6 \times 10–15$ m **A1**

Solution 4

(a)(ii) Worked steps

Know. Read one complete cycle off the time axis, then convert the period to an angular frequency.

Equation. $\omega = \dfrac{2\pi}{T}$.

Substitute. $T = 0.20\ \mu\text{s} = 0.20 \times 10^{-6}$ s, so $\omega = \dfrac{2\pi}{0.20 \times 10^{-6}}$.

Answer. $\omega = 3.1 \times 10^{7}$ rad s^{-1}.

Solution 4

Check. The microsecond is the trap. Leaving T as 0.20 gives 31 rad s^{-1}, a million times too small, and every later part inherits the error.

Mark scheme

- $\omega = 2\pi$ / (0.20 $\times$ 10–6)
- $=$ 3.1 $\times$ 107 rad s–1 **A1**

Solution 4

(a)(iii) Worked steps

Know. In simple harmonic motion the speed is greatest as the oscillator passes through the middle, and that maximum speed is ωx_0.
Equation. $v_0 = \omega x_0$.
Substitute. $v_0 = (3.1 \times 10^7) \times (3.6 \times 10^{-15})$.
Answer. $v_0 = 1.1 \times 10^{-7}$ m s^{-1}, as required.

Solution 4

Check. A “show that” is marked on the working, not the printed number. Write the equation, put both of your own earlier answers into it, and let the arithmetic land on the given value.

Mark scheme

- v0 $= \omega$x0 **C1**
- v0 $=$ 3.1 $\times$ 107 $\times$ 3.6 $\times$ 10–15 $=$ 1.1 $\times$ 10–7 m s–1 **A1**

Solution 4

(b)(i) Worked steps

Know. The current in a conductor is set by how many charge carriers there are, how fast they drift, and how much charge each carries. Here the drift speed is at its maximum, so the current is at its maximum too.

Equation. $I_0 = nAv_0e$.

Substitute. $I_0 = (8.5 \times 10^{28}) \times (4.3 \times 10^{-4}) \times (1.1 \times 10^{-7}) \times (1.60 \times 10^{-19})$.

Answer. $I_0 = 0.64$ A.

Solution 4

Check. A drift speed of 10^{-7} m s^{-1} giving a current of nearly an amp looks wrong until you notice *n*: there are around 10^{28} free electrons in every cubic metre of metal, so an almost motionless crowd still delivers a large current.

Mark scheme

- I0 $=$ nAv0e
- $=$ 8.5 $\times$ 1028 $\times$ 4.3 $\times$ 10–4 $\times$ 1.1 $\times$ 10–7 $\times$ 1.60 $\times$ 10–19 **C1**
- $=$ 0.64 A **A1**

Solution 4

(b)(ii) Worked steps

Know. The current is proportional to the drift velocity, and the drift velocity varies in step with the graph in part (a). So the current curve has the **same period and the same phase** as the displacement curve, only with a different scale on the axis.
Draw. Two complete cycles between $t = 0$ and $t = 0.40\ \mu$s, since one period is $0.20\ \mu$s.
Phase. Start at the top: $I = +I_0 = 0.64$ A at $t = 0$, falling to $-I_0$ at $t = 0.10\ \mu$s, back to $+I_0$ at $0.20\ \mu$s, and so on.

Solution 4

Check. Starting at zero would be a sine curve, which describes an oscillator released from the middle. This one is released from an extreme, so it is a cosine.

Mark scheme

- sketch: two cycles of sinusoidal curve of amplitude I0 and period 0.20 μs **B1**
- correct phase, with $\mathrm{I} = +\mathrm{I}0$ at $\mathrm{t} = 0$ **B1**

Solution 4

(b)(iii) Worked steps

Know. You have just drawn a cosine of amplitude I_0 and angular frequency ω, so write it down as an equation.
Equation. $I = I_0 \cos \omega t$.
Substitute. Use your own two answers: $I_0 = 0.64$ A from (b)(i) and $\omega = 3.1 \times 10^7$ rad s^{-1} from (a)(ii).
Answer. $I = 0.64 \cos\left(3.1 \times 10^7\, t\right)$, with I in A and t in s.

Solution 4

Check. The examiner marks the **form** first and the numbers second, and the numbers only have to match your own earlier answers. So an arithmetic slip earlier costs you nothing here, as long as you carry it forward consistently.

Mark scheme

- equation of form I $=$ I0 cos ωt **M1**
- value of I0 used matches answer to (b)(i) and value of ω used matches answer to (a)(ii)

.1 $\times$ 107 t) **A1**

Solution 4

(b)(iv) Worked steps

Know. The root-mean-square value of a sinusoidal current is the peak divided by $\sqrt{2}$.

Equation. $I_{\text{r.m.s.}} = \dfrac{I_0}{\sqrt{2}}$.

Substitute. $I_{\text{r.m.s.}} = \dfrac{0.64}{\sqrt{2}}$.

Solution 4

Answer. $I_{\text{r.m.s.}} = 0.45$ A.
Check. The r.m.s. value must be **smaller** than the peak. Multiplying by $\sqrt{2}$ instead of dividing gives 0.91 A, and the fact that it exceeds the maximum current is enough to spot the mistake.

Mark scheme

- Ir.m.s. $=$ I0 / $\sqrt{}$2
- $=$ 0.64 / $\sqrt{}$2
- $=$ 0.45 A **A1**

Question 7

9702/43 N23 ▪ Q7 ▪ 8 marks

A varying current I passes through a resistor of resistance R in the circuit shown in Fig. 7.1.

Fig. 7.2 shows the variation with time t of I.

The current has magnitude $2I_0$ when it is in the positive direction and I_0 when it is in the negative direction. The period of the variation of the current is T.

(a)(i) the current is in the negative direction **[1]**

(a)(ii) the current is in the positive direction. **[1]**

(b) On Fig. 7.3, sketch the variation of P with t between $t = 0$ and $t = 2.0T$. Label the power axis with an appropriate scale. **[3]**

(c)(i) the mean power $\langle P \rangle$ in the resistor **[1]**

(c)(ii) the root-mean-square (r.m.s.) current $I_{\text{r.m.s.}}$ in the resistor. **[2]**

Question 7

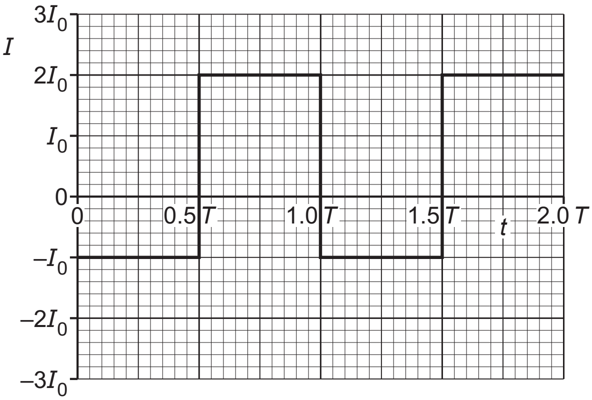


Fig. 7.2

Solution 7

(a)(i)

Mark scheme

- $P = I02R$ **A1**

(a)(ii)

Mark scheme

- $P = 4I02R$ **A1**

Solution 7

(b) Worked steps

Know. Power is I^2R, so the sign of the current does not matter but its **size** does. The current is constant within each half-cycle, so the power is constant too: the graph is a **square wave**, not a curve.

Levels. In the half-cycle where the current has magnitude I_0, the power is I_0^2R. In the half-cycle where it is $2I_0$, the power is $(2I_0)^2R = 4I_0^2R$, four times higher.

Draw. Horizontal lines at I_0^2R from $t = 0$ to $0.5T$ and again from T to $1.5T$; horizontal lines four times higher, at $4I_0^2R$, from $0.5T$ to T and from $1.5T$ to $2.0T$. Label the power axis with those two values.

Solution 7

Check. The power never reaches zero, and it never goes negative. A graph that dips below the axis has confused power with current.

Mark scheme

- sketch: square wave of period T, with P always non-zero **B1**
- horizontal lines, from 0 to 0.5T and from 1.0T to 1.5T, all at the same level that the scale indicates to be I02R **B1**
- horizontal lines, from 0.5T to 1.0T and from 1.5T to 2.0T, at a level that is four times higher than the lower lines **B1**

Solution 7

(c)(i) Worked steps

Know. The mean of a square wave that spends equal times at two levels is simply the average of those two levels.

Equation. $\langle P \rangle = \dfrac{I_0^2 R + 4 I_0^2 R}{2}$.

Answer. $\langle P \rangle = \dfrac{5}{2} I_0^2 R$.

Solution 7

Check. The answer must lie between the two levels, $I_0^2 R$ and $4I_0^2 R$, and $2.5\,I_0^2 R$ does. Averaging the **currents** instead, to get $1.5I_0$, and squaring that would give $2.25\,I_0^2 R$: close, but wrong, because squaring and averaging cannot be swapped round.

Mark scheme

- <P> = (5/2)I02R **A1**

Solution 7

(c)(ii) Worked steps

Know. The r.m.s. current is *defined* as the steady direct current that would dissipate the same mean power in the same resistor. So set the two powers equal.

Equation. $I_{\text{r.m.s.}}^2 R = \langle P \rangle$.

Substitute. $I_{\text{r.m.s.}}^2 R = \dfrac{5}{2} I_0^2 R$, and R cancels.

Answer. $I_{\text{r.m.s.}} = \sqrt{\dfrac{5}{2}}\, I_0 = 1.6\, I_0$.

Solution 7

Check. The $\frac{1}{\sqrt{2}}$ rule does **not** apply here: it belongs to a sine wave only, and this current is a square wave. Going back to the definition through the mean power is the method that works for any waveform at all.

Mark scheme

- $<P> = I_{r.m.s.}^2 R$ **C1**
- $I_{r.m.s.}^2 R = (5/2) I_0^2 R$
- $I_{r.m.s.} = \sqrt{(5/2)}\ I_0$ **A1**