Exercise walk-through

Characteristics of alternating currents ▪ 21.1

eXercise

You must be able to

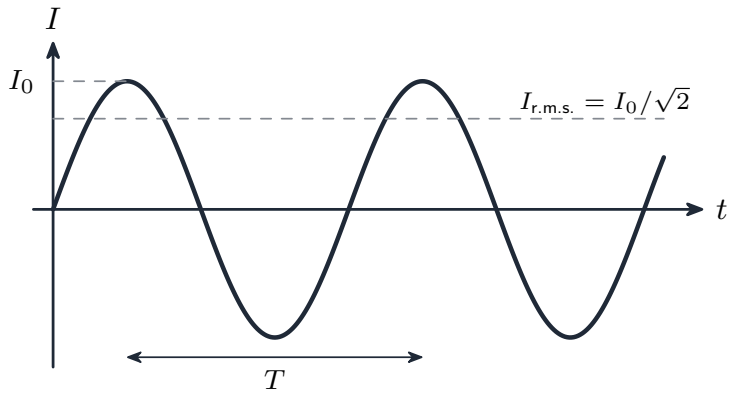
- use **period**, **frequency** and **peak value** for a.c.
- use $x = x_0 \sin \omega t$ and $I_{\text{r.m.s.}} = I_0/\sqrt{2}$
- use that the **mean power** is **half** the peak power

Method — Alternating current

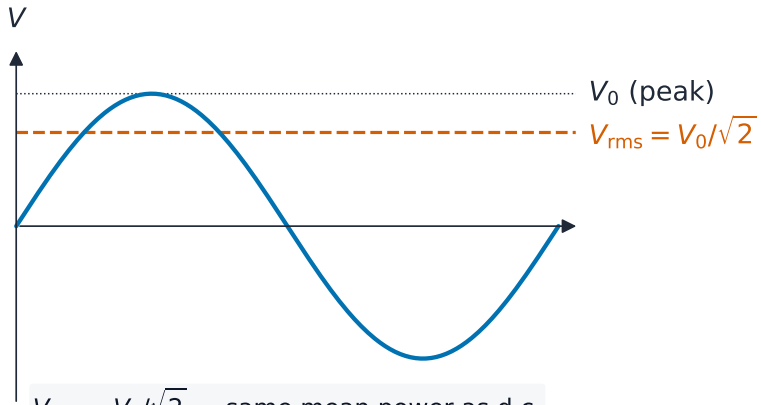
An a.c. follows $I = I_0 \sin \omega t$:

- **peak** I_0 (amplitude); **period** T; **frequency** $f = 1/T$; $\omega = 2\pi f$.

Method — Alternating current



Method — Alternating current



RMS values of an alternating current

Method — Reading the equation

Exam questions often **give** the a.c. as an equation, e.g. $V = 18\cos(40\pi t)$. Read off the **peak** ($V_0 = 18$ V) and the **angular frequency** ($\omega = 40\pi$ rad s^{-1}), then $T = \dfrac{2\pi}{\omega}$ and $f = \dfrac{1}{T}$.

Worked example. For $V = 18\cos(40\pi t)$: $\omega = 40\pi$, so $T = \dfrac{2\pi}{40\pi} = 0.050$ s and $f = 20$ Hz; $V_{\text{r.m.s.}} = 18/\sqrt{2} = 13$ V.

Method — r.m.s. and mean power

$$I_{\text{r.m.s.}} = \frac{I_0}{\sqrt{2}}, \qquad V_{\text{r.m.s.}} = \frac{V_0}{\sqrt{2}}.$$

The **mean power** in a resistor is **half** the peak power; $\langle P \rangle = I_{\text{r.m.s.}} V_{\text{r.m.s.}} = \dfrac{V_{\text{r.m.s.}}^2}{R}$.

Worked example. $V_0 = 340$ V: $V_{\text{r.m.s.}} = 340/\sqrt{2} = 240$ V.

Practice 2.1 ▪ 2 marks

Define the **peak value** and the **period** of an alternating current.

Solution 2.1

Method: Alternating current

Worked steps

Peak value — the maximum value in a cycle; **period** — the time for one complete cycle B1 B1.

Practice 2.2 ▪ 2 marks

An a.c. has a peak current of 5.0 A. Find the **r.m.s.** current.

Solution 2.2

Worked steps

$I_{\text{r.m.s.}} = I_0/\sqrt{2} = 5.0/\sqrt{2} = 3.5$ A **M1** **A1**.

Practice 2.3 ▪ 1 mark

State the relationship between the r.m.s. and peak **voltage**.

Solution 2.3

Worked steps

$V_{\text{r.m.s.}} = V_0/\sqrt{2}$ B1.

Practice 2.4 ▪ 1 mark

Mains frequency is 50 Hz. Find the **period**.

Solution 2.4

Worked steps

$T = 1/f = 1/50 = 0.020$ s B1.

Practice 2.5 ▪ 2 marks

An a.c. voltage is $V = 18\cos(40\pi t)$. State the **peak voltage** and the **angular frequency**.

Solution 2.5

Method: Reading the equation

Worked steps

peak $V_0 = 18$ V; $\omega = 40\pi$ rad s^{-1} (≈ 126 rad s^{-1}) B1 B1.

Exam-style 3.1 ▪ 1 mark

The r.m.s. value of a sinusoidal a.c. of peak I_0 is:

A I_0

B $I_0/2$

C $I_0/\sqrt{2}$

D $2I_0$

Solution 3.1

Method: Alternating current

A I_0

B $I_0/2$

C $I_0/\sqrt{2}$ ✓

D $2I_0$

Worked steps

C — $I_0/\sqrt{2}$.

Exam-style 3.2 ▪ 4 marks

A sinusoidal supply has a peak voltage of 340 V. Calculate (a) the r.m.s. voltage, (b) the mean power delivered to a $100\ \Omega$ resistor.

Solution 3.2

Method: r.m.s. and mean power

Worked steps

(a) $V_{\text{r.m.s.}} = 340/\sqrt{2} = 240$ V M1 A1. (b) $P = V_{\text{r.m.s.}}^2/R = 240^2/100 = 576$ W M1 A1.

Exam-style 3.3 ▪ 2 marks

Explain why the **mean power** in a resistor is **half** the peak power for a sinusoidal a.c.

Solution 3.3

Method: r.m.s. and mean power

Worked steps

The power $P = I^2R$ varies as $\sin^2$, whose **average over a cycle is** $\frac{1}{2}$ B1; so the mean power is half the peak power I_0^2R B1.

Exam-style 3.4 ▪ 2 marks

An a.c. has a period of 0.020 s. Find the **angular frequency** ω.

Solution 3.4

Method: Reading the equation

Worked steps

$\omega = 2\pi/T = 2\pi/0.020 = 3.1 \times 10^2$ rad s^{-1} M1 A1.

Exam-style 3.5 ▪ 2 marks

A mains voltage is $V = 340\sin(100\pi t)$, with V in volts and t in seconds.
(a) Show that the **period** is 0.020 s.
(b) Calculate the **r.m.s. voltage** and the **mean power** delivered to a $60\ \Omega$ heater.

Solution 3.5

Method: r.m.s. and mean power

Worked steps

(b) $V_{\text{r.m.s.}} = 340/\sqrt{2} = 240$ V **M1**; $\langle P \rangle = \dfrac{V_{\text{r.m.s.}}^2}{R} = \dfrac{240^2}{60} = 9.6 \times 10^2$ W **M1** **A1**.