Past-paper walk-through

Energy stored in a capacitor ▪ 19.2

eXercise

Questions in this set

- 9702/41 N25 Q6 ▪ 11 marks
- 9702/42 M24 Q4 ▪ 10 marks
- 9702/42 M23 Q5 ▪ 10 marks

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 6

9702/41 N25 ▪ Q6 ▪ 11 marks

Two parallel plate capacitors C_1 and C_2 are connected to a supply that has a potential difference (p.d.) V_S. The capacitors may be connected in series or in parallel.

The supply provides charge Q_S and the plates of the two capacitors acquire charges Q_1 and Q_2 respectively. The p.d.s across the plates of the capacitors are V_1 and V_2 respectively.

(a) Complete Table 6.1 to indicate how Q_S, Q_1 and Q_2 relate to each other, and how V_S, V_1 and V_2 relate to each other, for series and parallel connections of the capacitors to the supply.

Table 6.1

	relationship between charges	relationship between p.d.s
series		
parallel		

Question 6

[4]

Question 6

	relationship between charges	relationship between p.d.s
series		
parallel		

Question 6

9702/41 N25 ▪ Q6 ▪ 11 marks

Two parallel plate capacitors C_1 and C_2 are connected to a supply that has a potential difference (p.d.) V_S. The capacitors may be connected in series or in parallel.

The supply provides charge Q_S and the plates of the two capacitors acquire charges Q_1 and Q_2 respectively. The p.d.s across the plates of the capacitors are V_1 and V_2 respectively.

(b)(i) Calculate the p.d. across the capacitor. **[2]**

(b)(ii) Calculate the charge on the capacitor. **[2]**

(b)(iii) The capacitor is now connected in parallel with a capacitor of capacitance $180\ \mu$F that is initially uncharged.

Determine the total energy, in mJ, now stored in the two capacitors. **[3]**

Solution 6

(a) Mark scheme

- series charges:
- $QS = Q1 = Q2$ **B1**
- series p.d.s:
- $VS = V1 + V2$ **B1**
- parallel charges: $QS = Q1 + Q2$ **B1**
- parallel p.d.s:
- $VS = V1 = V2$ **B1**

Solution 6

(b)(i) Worked steps

- $E = \frac{1}{2}CV^2$, so $V = \sqrt{\dfrac{2E}{C}}$.
- $V = \sqrt{\dfrac{2 \times 19 \times 10^{-3}}{470 \times 10^{-6}}}$
- $V = 9.0$ V

Solution 6

Mark scheme

- E = ½ CV2 C1
- p.d. = [(2 × 19 × 10–3) / (470 × 10–6)]½
- = 9.0 V A1

Solution 6

(b)(ii)

Worked steps

Now that the p.d. is known, the charge follows from the definition of capacitance.

- $Q = CV = 470 \times 10^{-6} \times 9.0$
- $Q = 4.2 \times 10^{-3}$ C

Or in one step from the energy: $Q = \sqrt{2EC}$, which avoids carrying a rounded V.

Mark scheme

- E = Q2 / 2C or C = Q / V **C1**
- Q = (19 × 10–3 × 2 × 470 × 10–6)½ or
- Q = 470 × 10–6 × 9.0
- Q = 4.2 × 10–3 C **A1**

Solution 6

(b)(iii) Worked steps

Connecting a second capacitor in **parallel** to a charged one shares the charge, and the total charge is conserved.

- Total capacitance: $C = (470 + 180) \times 10^{-6} = 650\ \mu\text{F}$.
- The charge is unchanged at $Q = 4.2 \times 10^{-3}$ C from part (b)(ii).
- $E = \dfrac{Q^2}{2C} = \dfrac{(4.23 \times 10^{-3})^2}{2 \times 650 \times 10^{-6}}$
- $E = 0.014\ \text{J} = 14\ \text{mJ}$

Solution 6

Energy fell from 19 mJ to 14 mJ even though no charge was lost — the missing 5 mJ is dissipated in the connecting wires as the charge redistributes, which is the point the question is testing.

Mark scheme

- total charge unchanged **C1**
- total capacitance $= (470 + 180) \times 10–6$ (F) **C1**
- $E = Q2 / 2C = (4.23 \times 10–3)2 / (2 \times 650 \times 10–6)$ ($= 0.014$ J)
- $E = 14$ mJ **A1**

Question 4

9702/42 M24 ▪ Q4 ▪ 10 marks

Three capacitors are connected as shown in Fig. 4.1.

(a) Determine the total capacitance, in μF, of the network of three capacitors. **[2]**

(b) A capacitor of capacitance 45 μF is connected to a variable power supply initially set at 8.0V. The output of the power supply increases so that the potential difference (p.d.) across the capacitor increases to 9.6V.

Calculate the increase in energy ΔE stored in the capacitor. **[2]**

(c)(i) Complete the circuit in Fig. 4.2 by adding a capacitor to smooth the p.d. across the load resistor. **[1]**

(c)(ii) The variation with time t of the p.d. V of the smoothed output is shown in Fig. 4.3.

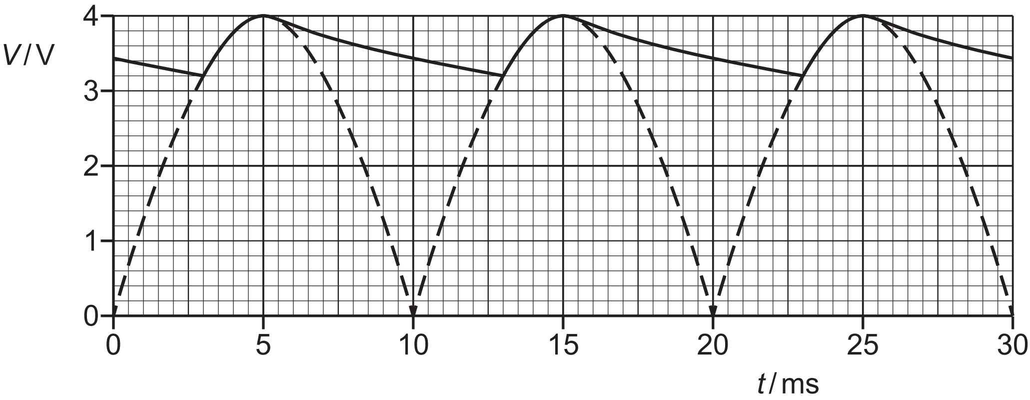
Determine the time constant, in ms, of the smoothing circuit. **[3]**

(d)(i) full-wave rectified **[1]**

Question 4

(d)(ii) half-wave rectified. **[1]**

Question 4



Solution 4

(a) Worked steps

Know. Work outwards from the inside of the network. Capacitors in **parallel** add directly; capacitors in **series** add as reciprocals. The rules are the opposite way round to resistors, which is where most marks are lost.

Parallel pair. The two parallel capacitors combine to $15 + 15 = 30\ \mu$F.

In series with the third. $\dfrac{1}{C} = \dfrac{1}{45} + \dfrac{1}{30}$.

Solution 4

Answer. $C = \left(\frac{1}{45} + \frac{1}{30}\right)^{-1} = 18\ \mu\text{F}$.

Check. A series combination must be **smaller** than either capacitor in it, and $18 < 30$, so the answer is the right side of both. Forgetting the final reciprocal leaves 0.056, an obvious flag.

Mark scheme

- combined capacitance of parallel capacitors $= 30$ (μF)
- total capacitance $= (1/45 + 1/30)^{-1}$
- $= 18\ \mu\text{F}$

Solution 4

(b) Worked steps

Know. "Increase in energy" means the difference between two stored energies, not the energy at some average voltage.
Equation. $E = \frac{1}{2}CV^2$, so $\Delta E = \frac{1}{2}C\left(V_2^2 - V_1^2\right)$.
Substitute. $\Delta E = \frac{1}{2} \times 45 \times 10^{-6} \times \left(9.6^2 - 8.0^2\right)$.
Answer. $\Delta E = 6.3 \times 10^{-4}$ J.

Solution 4

Check. Square each voltage **before** subtracting. Squaring the difference, $(9.6-8.0)^2 = 2.56$, gives 5.8×10^{-5} J: about a tenth of the true value, and wrong for the same reason $(a-b)^2 \neq a^2 - b^2$.

Mark scheme

- $E = \frac{1}{2}CV^2$
- $\Delta E = \frac{1}{2} \times 45 \times 10^{-6}(9.6^2 - 8.0^2)$
- $= 6.3 \times 10^{-4}$ J

Solution 4

(c)(i)

Mark scheme

- gaps in circuit closed and correct symbol for capacitor shown in parallel with load resistor

Solution 4

(c)(ii) Worked steps

Know. Between the peaks of the rectified supply the capacitor discharges through the load, so the falling part of each ripple is an exponential decay with time constant τ.

Read two points. Take both from **within one** falling section, for example $(5.0\ \text{ms},\ 4.0\ \text{V})$ and $(13.0\ \text{ms},\ 3.2\ \text{V})$, so $\Delta t = 8.0$ ms.

Equation. $V = V_0 \exp\left(-\dfrac{\Delta t}{\tau}\right)$.

Solution 4

Substitute. $3.2 = 4.0\exp\left(-\frac{8.0}{\tau}\right)$, so $\ln\left(\frac{3.2}{4.0}\right) = -\frac{8.0}{\tau}$.

Answer. $\tau = 36$ ms.

Solution 4

Check. Both read-offs must come from the **same** discharge, between one peak and the next. Taking V_0 from one ripple and V from a later one measures a decay that the recharging has already interrupted, and the time constant comes out far too large.

Mark scheme

- two correct pairs of values of t and V read off from within same discharge cycle, e.g. $(5.0, 4.0)$ and $(13.0, 3.2)$
- correct substitution of values of V, V_0 and Δt into $V = V_0 \exp(-\Delta t/\tau)$
- e.g. $3.2 = 4.0 \exp(-8.0/\tau)$
- $\tau = 36$ ms

Solution 4

(d)(i)

Mark scheme

- 8.0 W

(d)(ii)

Mark scheme

- 4.0 W

Question 5

9702/42 M23 ▪ Q5 ▪ 10 marks

A capacitor, a battery of electromotive force (e.m.f.) 12 V, a resistor R and a two-way switch are connected in the circuit shown in Fig. 5.1.

The switch is initially in position S. When the capacitor is fully charged, the switch is moved to position T so that the capacitor discharges. At time t after the switch is moved the charge on the capacitor is Q.

The variation with t of $\ln(Q/\mu\text{C})$ is shown in Fig. 5.2.

(a) Show that the capacitance of the capacitor is 1.5 μF. **[3]**

(b) Determine the resistance of R. **[3]**

(c) Calculate the energy stored in the capacitor at time $t = 0$. **[2]**

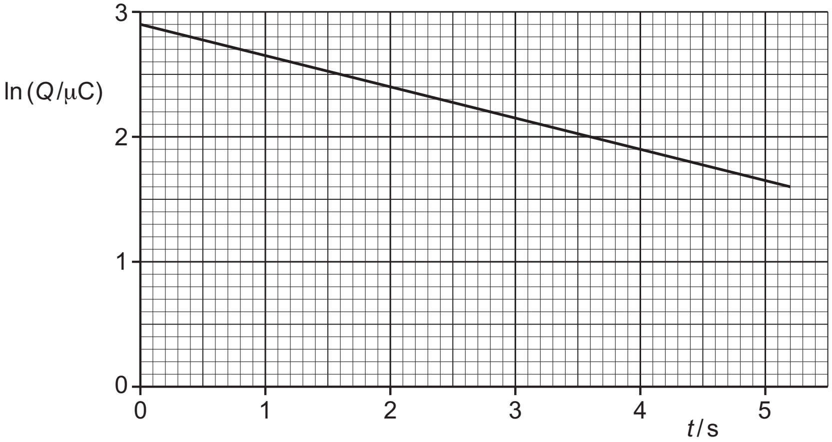
(d) A second identical resistor is now connected in parallel with R.

Question 5

The switch is initially in position S. When the capacitor is fully charged, the switch is moved to position T so that the capacitor discharges. At time t after the switch is moved the charge on the capacitor is Q.

On Fig. 5.2, sketch a line to show the variation of $\ln(Q/\mu\text{C})$ with t between time $t = 0$ and time $t = 5.0$ s. **[2]**

Question 5



Solution 5

(a) Worked steps

Know. The graph is of $\ln Q$ against t, so the **intercept** gives the charge at $t = 0$, which is the fully charged value.

Read the intercept. $\ln\left(\dfrac{Q}{\mu\text{C}}\right) = 2.9$ at $t = 0$, so $Q = e^{2.9} = 18\ \mu$C.

Equation. $C = \dfrac{Q}{V}$, with V the e.m.f. the capacitor was charged to, 12 V.

Substitute. $C = \dfrac{18.2 \times 10^{-6}}{12}$.

Solution 5

Answer. $C = 1.5 \times 10^{-6}$ F $= 1.5$ μF, as required.
Check. The vertical axis is a **logarithm**, so 2.9 is not a charge. Reading it as 2.9 μC gives 0.24 μF and loses all three marks.

Mark scheme

- from graph $\ln Q = 2.9$
- (so $Q = 18.2$ μC)
- $C = Q/V$
- $= 18.2/12 = 1.5$ μF

Solution 5

(b) Worked steps

Know. Taking logs of $Q = Q_0 \exp\left(-\frac{t}{RC}\right)$ gives $\ln Q = \ln Q_0 - \frac{t}{RC}$. That is the equation of a straight line whose gradient is $-\frac{1}{RC}$, which is why the graph was drawn this way.

Measure the gradient. From the line, gradient $= -0.25$ s^{-1}.

Equation. gradient $= -\frac{1}{RC}$, so $R = \frac{1}{0.25\ C}$.

Substitute. $R = \frac{1}{0.25 \times 1.5 \times 10^{-6}}$.

Answer. $R = 2.7 \times 10^{6}\ \Omega$.

Solution 5

Check. Use the capacitance in **farads**, not microfarads. And take the gradient from the drawn line over a large triangle, not from two crowded points: the whole reason for plotting logs is that a straight line can be measured accurately. **Mark scheme**

- gradient $= -0.25$
- gradient $= -1/RC$
- $R = 1/(0.25 \times 1.5 \times 10^{-6})$
- $= 2.7 \times 10^{6}\ \Omega$
- **or**
- $\frac{Q}{Q_0} = e^{-t/CR}$ or $\ln Q - \ln Q_0 = -\frac{t}{CR}$

Solution 5

- e.g. $\frac{4.95}{18.2} = e^{-5.2/(1.5\times10^{-6}R)}$ or $1.6 - 2.9 = 5.2/(1.5 \times 10^{-6}R)$
- $R = 2.7 \times 10^{6}\ \Omega$

Solution 5

(c) Worked steps

Know. At $t = 0$ the capacitor is fully charged, so use the values already found: $Q = 18\ \mu$C at $V = 12$ V.

Equation. $W = \frac{1}{2}QV$ (or equally $\frac{1}{2}CV^2$, or $\dfrac{Q^2}{2C}$ — all three give the same number).

Substitute. $W = \frac{1}{2} \times 18.2 \times 10^{-6} \times 12.$

Answer. $W = 1.1 \times 10^{-4}$ J.

Check. The factor of $\frac{1}{2}$ is not optional. QV alone would be the work done by the battery; half of that ends up in the capacitor and the other half is dissipated in the circuit resistance while charging. Mark scheme

Solution 5

- $W = \frac{1}{2}QV$
- $= \frac{1}{2} \times 18.2 \times 10^{-6} \times 12$
- $= 1.1 \times 10^{-4}$ J
- or $W = \frac{1}{2}CV^2$
- $= \frac{1}{2} \times 1.5 \times 10^{-6} \times 12^2$
- $= 1.1 \times 10^{-4}$ J
- or $W = \frac{1}{2}Q^2/C$
- $= \frac{1}{2} \times (18.2 \times 10^{-6})^2/1.5 \times 10^{-6}$
- $= 1.1 \times 10^{-4}$ J

Solution 5

(d) Worked steps

Know. Two identical resistors in parallel have **half** the resistance of one, so the time constant RC is halved and the capacitor discharges **twice as fast**.

Start point. The initial charge is unchanged: the capacitor is charged by the same 12 V battery through the same switch. So the new line starts at the same intercept, $(0,\ 2.9)$.

Gradient. The gradient is $-\dfrac{1}{RC}$, and R has halved, so the new gradient is **twice as steep** and still negative: about -0.50 s^{-1}.

Draw. A straight line from $(0,\ 2.9)$ falling twice as steeply as the original, drawn at least as far as $t = 5.0$ s.

Solution 5

Check. Two straight lines from the same intercept, one steeper than the other, is the whole answer. A curve would mean the discharge is no longer exponential, and putting the resistors in parallel does not change that.

Mark scheme

- straight line with different negative gradient starting from (0, 2.9)
- straight line between $t = 0$ and at least $t = 5.0$s with twice the gradient of the original line