Exercise walk-through

Electric potential ▪ 18.5

eXercise

You must be able to

- define **electric potential** and use $V = \dfrac{Q}{4\pi\varepsilon_0 r}$
- use $E_P = \dfrac{Qq}{4\pi\varepsilon_0 r}$ and $E = -$potential gradient

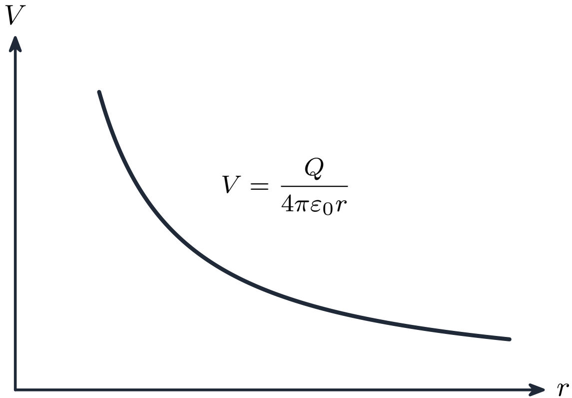
Method — Electric potential

Electric potential at a point is the **work done per unit positive charge** to bring a small test charge from **infinity** to that point (zero at infinity):

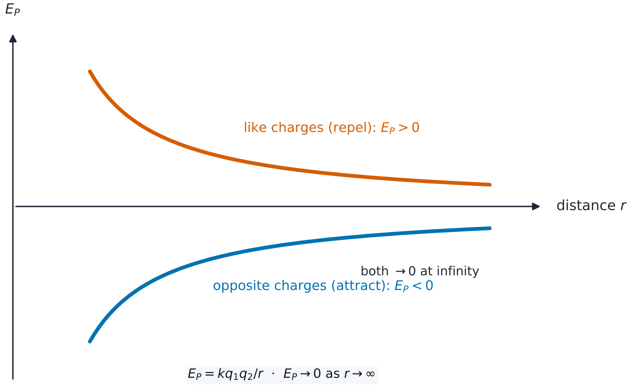
$$V = \frac{Q}{4\pi\varepsilon_0 r}, \qquad E_P = \frac{Qq}{4\pi\varepsilon_0 r}, \qquad E = -\frac{\Delta V}{\Delta r}.$$

For a **positive** charge $V > 0$ (falling to zero); for a **negative** charge $V < 0$.

Method — Electric potential



Method — Electric potential



Electric potential energy near a charge

Method — Adding potentials (a scalar)

Potential is a **scalar**, so the total potential from several charges is the **algebraic sum** (keep the signs) — no directions or vectors:

$$V = \sum \frac{Q_i}{4\pi\varepsilon_0 r_i}.$$

Worked example. Point P is 0.10 m from a $+4.0$ nC charge and 0.20 m from a $+6.0$ nC charge:

$$V = \frac{(9.0\times10^9)(4.0\times10^{-9})}{0.10} + \frac{(9.0\times10^9)(6.0\times10^{-9})}{0.20} = 360 + 270 = 630 \text{ V}.$$

Between a **positive** and a **negative** charge, the potentials have opposite sign, so there is a point where they **cancel** ($V = 0$): where $\dfrac{|Q_1|}{r_1} = \dfrac{|Q_2|}{r_2}$.

Practice 2.1 ▪ 1 mark

Define **electric potential**.

Solution 2.1

Method: Electric potential

Worked steps

The **work done per unit positive charge** in bringing a small test charge from **infinity** to that point **B1**.

Practice 2.2 ▪ 1 mark

Write the equation for the potential V due to a point charge.

Solution 2.2

Worked steps

$$V = \frac{Q}{4\pi\varepsilon_0 r} \quad \boxed{\textbf{B1}}.$$

Practice 2.3 ▪ 1 mark

Write the equation for the **electric potential energy** of two point charges.

Solution 2.3

Method: Electric potential

Worked steps

$$E_\mathrm{P} = \frac{Qq}{4\pi\varepsilon_0 r} \quad \boxed{\textbf{B1}}.$$

Practice 2.4 ▪ 1 mark

State the relationship between the **field** E and the **potential gradient**.

Solution 2.4

Method: Adding potentials (a scalar)

Worked steps

The field is the **negative of the potential gradient**: $E = -\dfrac{\Delta V}{\Delta r}$ **B1**.

Exam-style 3.1 ▪ 1 mark

The electric potential at **infinity** is:

A a maximum

B zero

C always negative

D infinite

Solution 3.1

Method: Electric potential

A a maximum

B zero ✓

C always negative

D infinite

Worked steps

B — zero.

Exam-style 3.2 ▪ 3 marks

Calculate the electric potential at 0.050 m from a $+3.0$ nC point charge. ($\frac{1}{4\pi\varepsilon_0} = 9.0 \times 10^9$.)

Solution 3.2

Worked steps

$$V = \frac{(9.0 \times 10^{9})(3.0 \times 10^{-9})}{0.050} \quad \textbf{M1}\,\textbf{M1} = 540\ \mathrm{V} \quad \textbf{A1}.$$

Exam-style 3.3 ▪ 2 marks

Calculate the electric potential energy of a $+2.0$ nC and a $+3.0$ nC charge placed 4.0 cm apart.

Solution 3.3

Method: Electric potential

Worked steps

$$E_{\mathsf{P}} = \frac{(9.0 \times 10^{9})(2.0 \times 10^{-9})(3.0 \times 10^{-9})}{0.040} \quad \textbf{M1} \quad = 1.35 \times 10^{-6}\ \mathsf{J} \quad \textbf{A1}.$$

Exam-style 3.4 ▪ 1 mark

State **one** difference between the electric potential near a **positive** charge and near a **negative** charge.

Solution 3.4

Method: Electric potential

Worked steps

Near a **positive** charge the potential is **positive**; near a **negative** charge it is **negative** B1.

Exam-style 3.5 ▪ 1 mark

A $+5.0$ nC charge is at point A and a -2.0 nC charge is at point B, 0.21 m away on the line AB.
(a) Explain why the total potential is found by **adding** the two potentials, including their signs.
(b) Find the distance from A, on the line AB, where the total potential is **zero**.

Solution 3.5

Method: Adding potentials (a scalar)

Worked steps

(b) $V = 0$ where the magnitudes cancel: $\dfrac{5.0}{x} = \dfrac{2.0}{0.21 - x}$ **M1**; $5.0(0.21 - x) = 2.0x \Rightarrow 1.05 = 7.0x$ **M1**; $x = 0.15$ m from A **A1**.

Exam-style 3.6 ▪ 2 marks

Two point charges are fixed on a straight line. Charge A, of $+4.0$ pC, is at the origin; charge B, of -1.0 pC, is at $x = +40$ pm. Take $\dfrac{1}{4\pi\varepsilon_0} = 8.99 \times 10^9$ m F^{-1}.

(a) **Define** the **electric potential** at a point.

(b) **Show that** the electric potential due to charge A alone, at $x = 10$ pm, is about 3.6 V.

(c) **Calculate**, to two significant figures, the **total** potential at $x = 10$ pm due to both charges.

(d) **Determine** the position on the line **between** the two charges at which the total electric potential is **zero**.

(e) **State and explain** whether the electric **field** is also zero at that position.

Solution 3.6

Method: Adding potentials (a scalar)

Worked steps

(a) The **work done per unit positive charge** in bringing a small test charge from **infinity** to that point B1 B1.

(b) $V = \dfrac{Q}{4\pi\varepsilon_0 r} = \dfrac{8.99 \times 10^9 \times 4.0 \times 10^{-12}}{10 \times 10^{-12}}$ M1 $= 3.6$ V A1.

(c) B is 30 pm away, so $V_B = \dfrac{8.99 \times 10^9 \times (-1.0 \times 10^{-12})}{30 \times 10^{-12}} = -0.30$ V M1 M1.

Potential is a **scalar**, so the two simply add: $V = 3.6 - 0.30 = 3.3$ V A1.

(d) Let the point be x from A, so it is $(40 - x)$ from B: $\dfrac{4.0}{x} = \dfrac{1.0}{40 - x}$ M1. Then

Solution 3.6

$4.0(40 - x) = x$, so $160 = 5.0x$ M1 and $x = 32$ pm from A A1.
(e) **No** B1. Potential is a scalar and cancels there, but the fields of the two charges at that point both point in the **same direction** (away from the positive charge and towards the negative one), so they add rather than cancel B1.