Exercise walk-through

Electric field of a point charge ▪ 18.4

eXercise

You must be able to

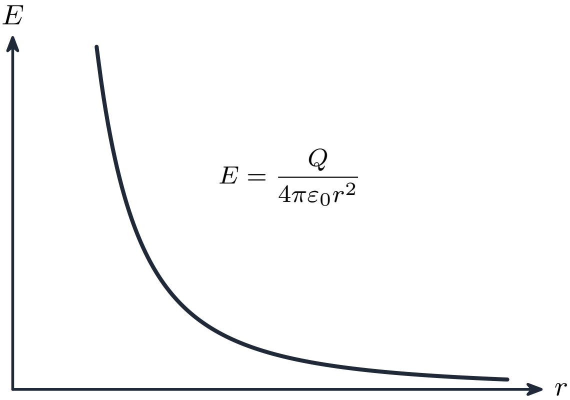
- use $E = \dfrac{Q}{4\pi\varepsilon_0 r^2}$
- compare electric and gravitational fields

Method — Field of a point charge

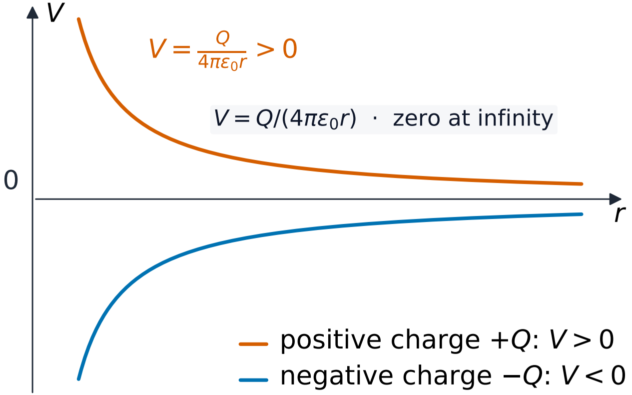
$$E = \frac{Q}{4\pi\varepsilon_0 r^2}.$$

It points **out** from a positive charge and **in** to a negative one, and falls as an **inverse square**:
This is just like the **gravitational** field $g = GM/r^2$ — except gravity is **always attractive**, while the electric field can be attractive or repulsive.

Method — Field of a point charge



Method — Field of a point charge



The radial field of a point charge

Practice 2.1 ▪ 1 mark

Write the equation for the field strength E due to a **point charge**.

Solution 2.1

Method: Field of a point charge

Worked steps

$$E = \frac{Q}{4\pi\varepsilon_0 r^2} \quad \boxed{\mathbf{B1}}.$$

Practice 2.2 ▪ 1 mark

State how E varies with distance r from a point charge.

Solution 2.2

Method: Field of a point charge

Worked steps

$E \propto 1/r^2$ (inverse-square) **B1**.

Practice 2.3 ▪ 2 marks

Find the field strength at 0.10 m from a $+5.0$ nC point charge. ($\frac{1}{4\pi\varepsilon_0} = 9.0 \times 10^9$.)

Solution 2.3

Method: Field of a point charge

Worked steps

$$E = \frac{(9.0 \times 10^9)(5.0 \times 10^{-9})}{(0.10)^2} = 4.5 \times 10^3 \text{ N C}^{-1} \quad \textbf{M1}\,\textbf{A1}.$$

Exam-style 3.1 ▪ 1 mark

Both the electric field of a point charge and the gravitational field of a point mass vary with distance as:

A $1/r$

B $1/r^2$

C r

D r^2

Solution 3.1

Method: Field of a point charge

A $1/r$

B $1/r^2$ ✓

C r

D r^2

Worked steps

B — $1/r^2$.

Exam-style 3.2 ▪ 3 marks

Calculate the electric field strength at 2.0 cm from a point charge of $+8.0$ nC.

Solution 3.2

Method: Field of a point charge

Worked steps

$$E = \frac{(9.0 \times 10^9)(8.0 \times 10^{-9})}{(0.020)^2} \quad \text{M1} \; \text{M1} \quad = 1.8 \times 10^5 \text{ N C}^{-1} \quad \text{A1}.$$

Exam-style 3.3 ▪ 2 marks

State **one** similarity and **one** difference between an electric field and a gravitational field.

Solution 3.3

Method: Field of a point charge

Worked steps

Similarity: both follow an **inverse-square** law (radial field around a point source). Difference: the electric force can be **attractive or repulsive**, but gravity is **always attractive** B1 B1.

Exam-style 3.5 ▪ 2 marks

An isolated solid metal sphere of radius R carries a positive charge $+Q$. Point Q lies outside the sphere, a distance $2R$ from its centre.

(a) **Explain** why the electric potential everywhere near an isolated **positive** charge is positive.

(b) **State** the direction of the electric field at point Q, and **state** how the field inside the metal sphere compares with it.

(c) On the axes below, **sketch** the variation of the electric field strength E with distance x from the centre of the sphere, for x from 0 to $4R$.

(d) The field strength at the surface of the sphere is E_0. **Determine**, in terms of E_0, the field strength at point Q.

(e) A second identical sphere carrying the same charge is now placed with its centre $6R$ from the centre of the first. **Determine** the position on the line joining their centres at

Exam-style 3.5 ▪ 2 marks

which the resultant field is zero, and **state** what the potential is there compared with the potential at the surface of either sphere.

Solution 3.5

Method: Field of a point charge

Worked steps

(a) Potential is the work done per unit positive charge bringing a test charge from infinity **B1**; a positive charge **repels** the test charge, so external work must be done **on** it to bring it closer, making the potential positive everywhere around the charge **B1**.

(b) At Q the field points **radially away** from the sphere, i.e. directly outwards along the line from the centre **B1**. Inside the metal the field is **zero** — the charge resides on the surface and the interior is field-free **B1**.

(c) $E = 0$ from $x = 0$ to $x = R$, drawn along the axis **B1**; a **discontinuous jump** to

Solution 3.5

the maximum E_0 at $x = R$ **B1**; then an inverse-square decay for $x > R$, falling to $E_0/4$ at $2R$ and $E_0/16$ at $4R$ **B1**.

(d) Outside the sphere it behaves as a point charge at its centre, so $E \propto \dfrac{1}{x^2}$ **M1**; at $2R$ the field is $\dfrac{E_0}{4}$ **A1**.

(e) By symmetry the fields cancel at the **midpoint**, $3R$ from each centre **M1** **A1**. The potentials, being scalars, **add** there, so the potential is not zero — it is smaller than at either surface but still positive **B1**.