Past-paper walk-through

Damped and forced oscillations, resonance ▪ 17.3

eXercise

Questions in this set

- 9702/42 N25 Q5 ▪ 10 marks
- 9702/44 J25 Q7 ▪ 8 marks
- 9702/43 N23 Q4 ▪ 9 marks

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 5

9702/42 N25 ▪ Q5 ▪ 10 marks

A steel ball on the end of a thin string oscillates with small oscillations, as shown in Fig. 5.1.

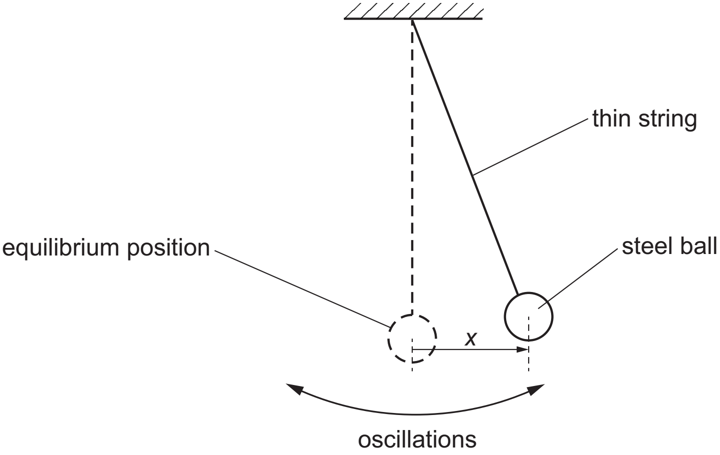
(a)(i) Explain how Fig. 5.2 shows that the oscillations of the ball are simple harmonic. **[2]**

(a)(ii) Determine the period T of the oscillations. **[3]**

(b)(i) State what is meant by damping. **[2]**

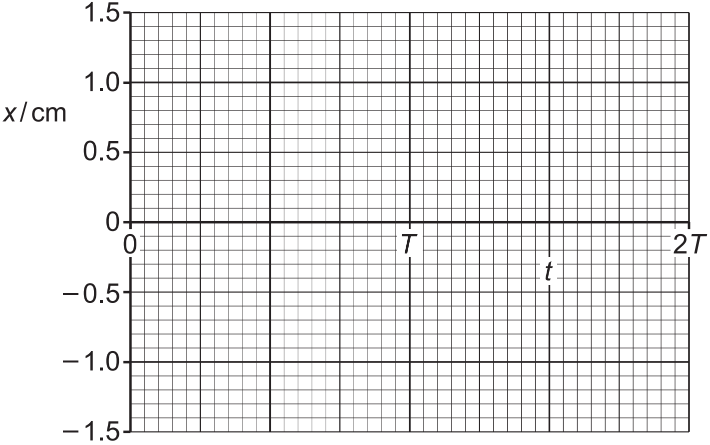
(b)(ii) On Fig. 5.3, sketch a possible variation of the displacement x of the ball with t between $t = 0$ and $t = 2T$. **[3]**

Question 5

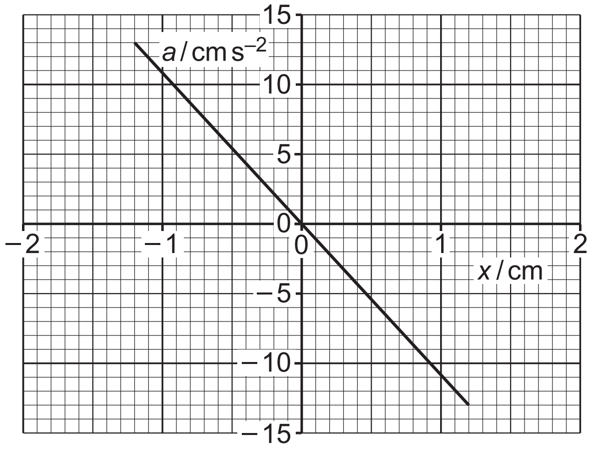


thin string
equilibrium position
steel ball
x
oscillations

Question 5



Question 5



Solution 5

(a)(i)

Mark scheme

- straight line through the origin shows that a is proportional to x **B1**
- negative gradient shows that a is always in the opposite direction to x **B1**

Solution 5

(a)(ii) Worked steps

Use the relation that defines simple harmonic motion, applied to the two maxima.

- For SHM, $a = -\omega^2 x$, so the graph's gradient gives ω^2 and the maxima satisfy $a_0 = \omega^2 x_0$.
- Read x_0 and a_0 off Fig. 5.2 and rearrange: $\omega = \sqrt{\dfrac{a_0}{x_0}}$.
- Then $T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{x_0}{a_0}}$.
- With the values from the graph, $T = 2\pi\sqrt{\dfrac{1.2}{13}} = 1.9$ s.

Solution 5

Keep x_0 and a_0 in **matching** units — the ratio is what matters, so reading both off the same graph in the units printed on its axes is safest.

Mark scheme

- a0 $=$ ω2x0 **C1**
- ω $=$ 2π / T **C1**
- T $=$ 2π $\sqrt{}$(x0 / a0)
- $=$ 2π $\sqrt{}$ (1.2 / 13)
- $=$ 1.9 s **A1**

Solution 5

(b)(i)

Mark scheme

- loss of energy of oscillations **B1**
- due to resistive force(s) **B1**

Solution 5

(b)(ii) Worked steps

Do not sketch a decaying wave here. Read the three things the mark scheme asks for, and each one rules an ordinary damped oscillation out:

1. The curve **starts at** $x = 1.2$ **cm** when $t = 0$ — the amplitude found in part (a), because the ball is released from its maximum displacement.
2. It stays **entirely on one side of the** t**-axis** for the whole interval 0 to $2T$ — so the ball never passes through the equilibrium position at all.
3. Both the **magnitude of** x **and the magnitude of the gradient decrease continuously** — it creeps back towards zero, more and more slowly, and never turns round.

Solution 5

A curve that never crosses the axis is not lightly damped: this is **heavy (over) damping**. So the sketch is a single falling curve from 1.2 cm that flattens towards the t-axis and approaches it without touching or crossing it.

Solution 5

Two ways to lose all three marks at once: drawing a sine wave inside a decaying envelope (that crosses the axis every half period), and starting the curve at the origin. The displacement is a maximum at $t = 0$, not zero.

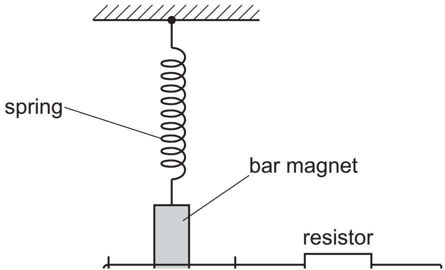
Mark scheme

- line starting from x = ±1.2 cm at t = 0 **B1**
- line starting from non-zero value of x from t = 0 to t = 2T that is entirely either above or below the t-axis **B1**
- curve from t = 0 starting from non-zero x value, with both magnitude of x value and magnitude of gradient continuously decreasing **B1**

Question 7

9702/44 J25 ▪ Q7 ▪ 8 marks

7 A bar magnet is suspended from a spring. One pole of the magnet oscillates freely in a coil of wire, as shown in Fig. 7.1.



Question 7

Fig. 7.1

The switch S is initially open.

(a) The switch S is now closed. As a result, the oscillations of the magnet are lightly damped.

(i) State what is meant by damping.

Question 7

(ii) Describe what is observed to indicate that the damping is light.

Question 7

(iii) By reference to electromagnetic induction and to conservation of energy, explain why the oscillations are damped.

Question 7

(b) The procedure in **(a)** is repeated after replacing the resistor with one of greater resistance.

Suggest, with a reason, the effect of this change on the oscillations.

Question 7

[Total: 8]

Solution 7

(a)(i)

Mark scheme

- loss of energy (of oscillations)
- due to resistive forces

Solution 7

(a)(ii)

Mark scheme

- amplitude (of oscillations) decreases gradually
- or
- oscillations continue (for several periods)

Solution 7

(a)(iii) Mark scheme

- cutting of (magnetic) flux causes induced e.m.f. (in coil)
- or
- induced e.m.f. causes current (in resistor / circuit)
- current causes dissipation of thermal energy in resistor
- or
- current in resistor causes dissipation of thermal energy
- thermal energy comes from energy of oscillations

Solution 7

(b) Worked steps

Follow the chain from resistance to amplitude — every link is one step, and the marks are for naming two of them.

- The induced e.m.f. is unchanged (it depends on the rate of change of flux linkage, not on the circuit), so a **higher resistance means a lower induced current**.
- By Lenz's law that current opposes the motion, so a smaller current means a **smaller resistive force** on the magnet — and, equivalently, less energy dissipated as I^2R per second.
- Less energy removed each cycle means the **oscillations are less damped**: a smaller decrease in amplitude per period, and the motion continues for longer.

Solution 7

Start from the **current**, not from the e.m.f. Saying "the e.m.f. is smaller" is wrong — the e.m.f. is set by the magnet's motion, and it is the current the resistance controls.

Mark scheme

- (for any given e.m.f.) current is lower so thermal energy dissipated at a lower rate
- or
- (for any given e.m.f.) current is lower so the resistive force is lower
- oscillations are less damped
- or
- smaller decrease in amplitude of oscillations (in each period)

Solution 7

- or
- oscillations continue for longer
- 9702/44
- May/June 2025

Question 4

9702/43 N23 ▪ Q4 ▪ 9 marks

A heavy metal sphere of mass 0.81 kg is suspended from a string. The sphere is undergoing small oscillations from side to side, as shown in Fig. 4.1.

The oscillations of the sphere may be considered to be simple harmonic with amplitude 0.036 m and period 3.0 s.

(a) State what is meant by simple harmonic motion. **[2]**

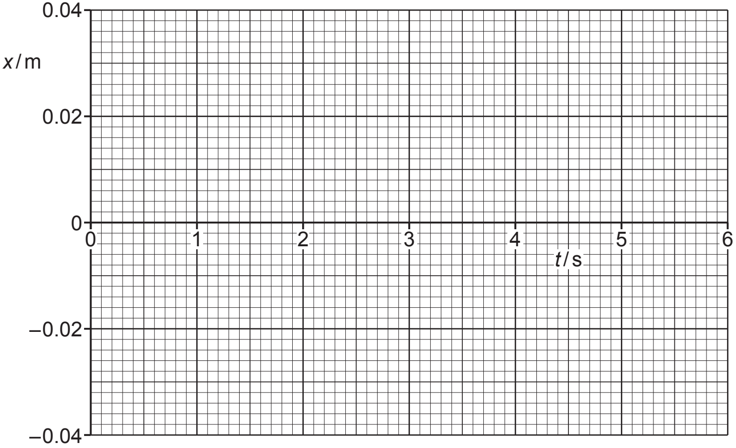
(b)(i) the angular frequency of the oscillations **[2]**

(b)(ii) the total energy of the oscillations. **[2]**

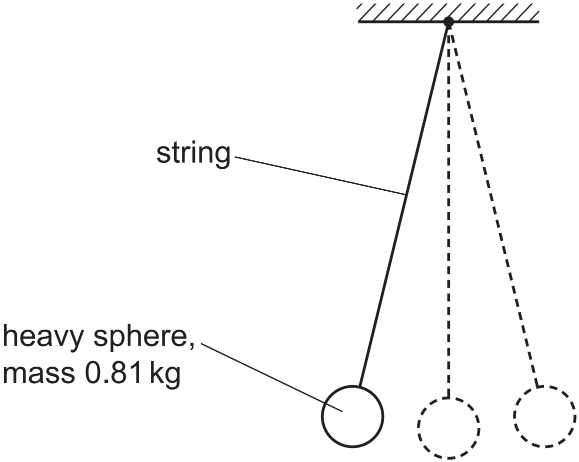
(c) The suspended sphere is now lowered into water. The sphere is given a sideways displacement of $+0.036$ m from its equilibrium position and is then released at time $t = 0$. The water causes the motion of the sphere to be critically damped.

On Fig. 4.2, sketch the variation of the displacement x of the sphere from its equilibrium position with t from $t = 0$ to $t = 6.0$ s. **[3]**

Question 4



Question 4



Solution 4

(a)

Mark scheme

- (motion in which) acceleration is (directly) proportional to displacement **B1**
- (motion in which):
- acceleration is (always) in the opposite direction to displacement or acceleration is (always) directed towards a fixed point **B1**

Solution 4

(b)(i)

Mark scheme

- $\omega = 2\pi$ / T **C1**
- $\omega = 2\pi$ / 3.0
- = 2.1 rad s–1 **A1**

Solution 4

(b)(ii) Worked steps

Know. The total energy of a simple harmonic oscillator equals its kinetic energy as it passes through the middle, where the speed is ωx_0.

Angular frequency first. $\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{3.0} = 2.1$ rad s^{-1}.

Equation. $E = \frac{1}{2} m\omega^2 x_0^2$.

Solution 4

Substitute. $E = \frac{1}{2}(0.81)(2.1)^2(0.036)^2$.
Answer. $E = 2.3 \times 10^{-3}$ J.
Check. Both ω and x_0 are squared. Squaring only one of them is the common slip, and it changes the answer by a factor of tens. The energy is also constant throughout the undamped motion, so it does not matter where in the swing the question is asking about.

Mark scheme

- E = ½mω2x02 **C1**
- = ½ × 0.81 × 2.12 × 0.0362
- = 2.3 × 10–3 J **A1**

Solution 4

(c) Worked steps

Know. Critical damping is the boundary case: the sphere returns to the middle in the shortest possible time and does **not** overshoot. There is no oscillation at all, so the graph crosses the axis at most once.

Start. The line begins at $(0,\ 0.036\ \text{m})$, the displacement at which it was released.

Shape. A smooth curve falling steadily to zero, with no sudden kinks and no part of it returning to ± 0.036 m. As it settles the gradient flattens to zero.

Timing. The displacement should reach zero somewhere between about 0.75 s and 3.0 s, that is, in less than one period of the undamped swing.

Solution 4

Check. A wavy line of shrinking amplitude is **light** damping, not critical, and it is the error most often drawn here. A line that takes the whole 6.0 s to settle is heavy damping. Critical damping is the fastest return without an overshoot, which is exactly why car suspensions and moving-coil meters are designed to be close to it.

Mark scheme

- sketch: line starting at (0, 0.036) and not reaching $x = \pm\ 0.036$ m at any other time **B1**
- smooth curve, with no sudden changes in gradient, showing continuously decreasing magnitude of x from maximum displacement at $t = 0$ to final displacement of zero where the gradient is also zero **B1**
- displacement reaches final value of zero between $t = 0.75$ s and $t = 3.0$ s at the latest **B1**