Exercise walk-through

Simple harmonic oscillations ▪ 17.1

eXercise

You must be able to

- use the SHM condition $a = -\omega^2 x$ and $x = x_0 \sin \omega t$
- use $v = \pm\omega\sqrt{x_0^2 - x^2}$ and $v_{\max} = \omega x_0$
- interpret the displacement, velocity and acceleration graphs

Method — What SHM is

A particle moves with **SHM** when its acceleration is **proportional to displacement** and **directed back** to equilibrium:

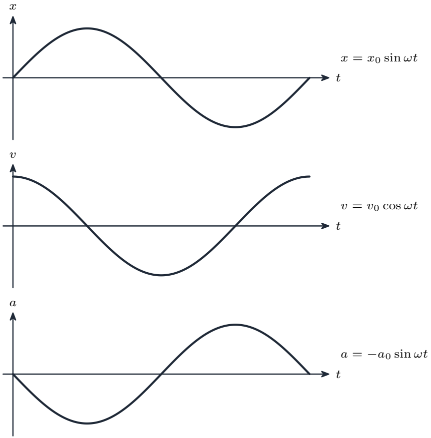
$$a = -\omega^2 x, \qquad x = x_0 \sin \omega t, \qquad T = \frac{2\pi}{\omega}.$$

Method — Velocity and the graphs

$$v = v_0 \cos \omega t, \qquad v = \pm\omega\sqrt{x_0^2 - x^2}, \qquad v_{\max} = \omega x_0, \qquad a_{\max} = \omega^2 x_0.$$

Displacement and acceleration are **antiphase**; velocity leads displacement by a **quarter cycle**.

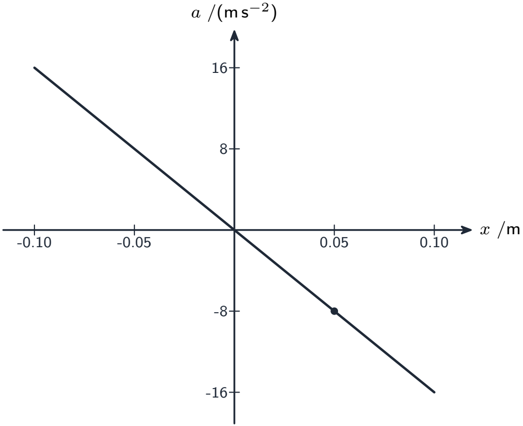
Method — Velocity and the graphs



Method — The acceleration–displacement graph

Because $a = -\omega^2 x$, a graph of a against x is a **straight line through the origin** with a **negative gradient** of magnitude ω^2:
This is the **test for SHM**: $a \propto x$ (straight line through the origin) and **opposite in direction** (negative gradient). Read the gradient to find ω: gradient $= -\omega^2$, so $\omega = \sqrt{|\text{gradient}|}$.
Worked example. The line above passes through $(0.05\ \text{m}, -8\ \text{m s}^{-2})$, so gradient $= -8/0.05 = -160\ \text{s}^{-2} = -\omega^2$; thus $\omega = \sqrt{160} = 12.6$ rad s^{-1} and $T = 2\pi/\omega = 0.50$ s.

Method — The acceleration–displacement graph



Practice 2.1 ▪ 2 marks

State the **two** conditions for simple harmonic motion.

Solution 2.1

Worked steps

Acceleration is **proportional to displacement** from a fixed point B1, and **directed towards** that point (opposite to displacement) B1.

Practice 2.2 ▪ 1 mark

Define the **amplitude** of an oscillation.

Solution 2.2

Worked steps

The **maximum displacement** from the equilibrium position **B1**.

Practice 2.3 ▪ 2 marks

A particle has $\omega = 5.0$ rad s^{-1} and amplitude 0.040 m. Find its **maximum speed**.

Solution 2.3

Worked steps

$v_{\max} = \omega x_0 = 5.0 \times 0.040 = 0.20 \text{ m s}^{-1}$ M1 A1.

Practice 2.4 ▪ 1 mark

State the **phase relationship** between displacement and acceleration in SHM.

Solution 2.4

Worked steps

They are **antiphase** (phase difference π / $180°$) **B1**.

Exam-style 3.1 ▪ 1 mark

In simple harmonic motion, the acceleration is:

A proportional to displacement and in the same direction

B proportional to displacement and in the opposite direction

C constant

D always zero

Solution 3.1

- **A** proportional to displacement and in the same direction
- **B** proportional to displacement and in the opposite direction ✓
- **C** constant
- **D** always zero

Worked steps

B — proportional to displacement and opposite in direction.

Exam-style 3.2 ▪ 4 marks

A mass on a spring oscillates with period 0.50 s and amplitude 0.030 m. Find (a) the angular frequency, (b) the maximum speed, (c) the maximum acceleration.

Solution 3.2

Method: The acceleration–displacement graph

Worked steps

(a) $\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{0.50} = 12.6$ rad s^{-1} **M1** **A1**. (b) $v_{\max} = \omega x_0 = 12.6 \times 0.030 = 0.38$ m s^{-1} **A1**. (c) $a_{\max} = \omega^2 x_0 = 12.6^2 \times 0.030 = 4.7$ m s^{-2} **A1**.

Exam-style 3.3 ▪ 3 marks

A particle in SHM has amplitude 0.10 m and $\omega = 8.0$ rad s^{-1}. Find its **speed** when $x = 0.060$ m.

Solution 3.3

Worked steps

$v = \omega\sqrt{x_0^2 - x^2} = 8.0\sqrt{0.10^2 - 0.060^2}$ **M1** $= 8.0\sqrt{0.0064}$ **M1** $= 8.0 \times 0.080 =$ $0.64\ \mathrm{m\ s^{-1}}$ **A1**.

Exam-style 3.4 ▪ 3 marks

Sketch the **displacement–time** and **acceleration–time** graphs for SHM, and state their phase relationship.

Solution 3.4

Method: Velocity and the graphs

Worked steps

Displacement is a **sine** (or cosine) curve; acceleration is the **same shape inverted** (negative) B1 B1; they are **antiphase** ($180°$ out of phase) B1.

Exam-style 3.5 ▪ 2 marks

An object oscillates. A graph of its acceleration a against displacement x is a straight line through the origin passing through $(0.05\ \text{m}, -8.0\ \text{m s}^{-2})$.

(a) Explain how this graph shows the motion is **simple harmonic**.

(b) Use the **gradient** to find the angular frequency ω and the period T.

Solution 3.5

Method: The acceleration–displacement graph

Worked steps

(b) gradient $= \dfrac{-8.0}{0.05} = -160\ \text{s}^{-2} = -\omega^2$ **M1**; $\omega = \sqrt{160} = 12.6\ \text{rad s}^{-1}$ **A1**;
$T = \dfrac{2\pi}{\omega} = 0.50\ \text{s}$ **A1**.

Exam-style 3.6 ▪ 2 marks

A small ball hangs on a spring and is set oscillating vertically. A motion sensor records its displacement, and the graph obtained is a smooth curve whose successive maximum displacements are equal, with a maximum displacement of x_0 and 8 complete oscillations in 6.4 s.

(a) A second graph plots the ball's **acceleration** against its **displacement** and gives a **straight line through the origin with a negative gradient**. **Explain** how this shows that the motion is simple harmonic.

(b) **Determine** the period T and the angular frequency ω of the oscillations.

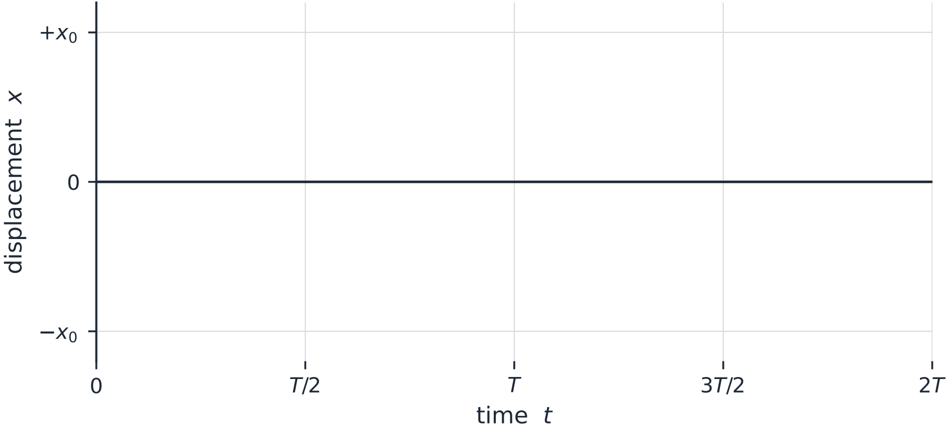
(c) The magnitude of the gradient of that acceleration-displacement line is 61 s^{-2}. **Show that** this is consistent with your value of ω.

(d) **State** what is meant by **damping**.

Exam-style 3.6 ▪ 2 marks

(e) The ball is now made to oscillate in a beaker of oil, and is released from the same displacement x_0. On the axes below, **sketch** a possible variation of its displacement with time from $t = 0$ to $t = 2T$.

Exam-style 3.6 ▪ 2 marks



Solution 3.6

Method: The acceleration–displacement graph

Worked steps

(a) A straight line through the origin shows $a \propto x$ B1; the negative gradient shows the acceleration is always directed **opposite** to the displacement, i.e. towards the equilibrium position — which is the definition of simple harmonic motion B1.

(b) $T = \dfrac{6.4}{8} = 0.80$ s M1 A1; $\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{0.80} = 7.9$ rad s^{-1} A1.

(c) For SHM $a = -\omega^2 x$, so the magnitude of the gradient is ω^2 M1: $\omega = \sqrt{61} = 7.8$ rad s^{-1}, which agrees with (b) A1.

(d) Damping is the loss of energy from an oscillating system to its surroundings, usually through **resistive forces** B1, so the **amplitude decreases** with time B1.

Solution 3.6

(e) A curve starting at $+x_0$ **B1**, oscillating with the **same period** T so that it still completes two cycles by $2T$ **B1**, with each maximum displacement **smaller** than the one before, decaying smoothly **B1**.