Exercise walk-through

Kinetic theory of gases ▪ 15.3

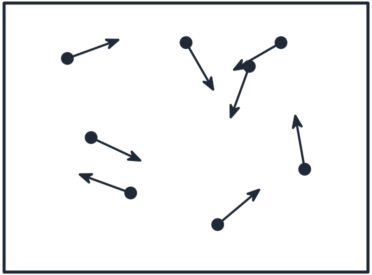
eXercise

You must be able to

- state the **assumptions** of the kinetic theory and explain gas pressure
- use $pV = \frac{1}{3}Nm\langle c^2\rangle$ and the **r.m.s. speed**
- use mean translational KE $= \frac{3}{2}kT$

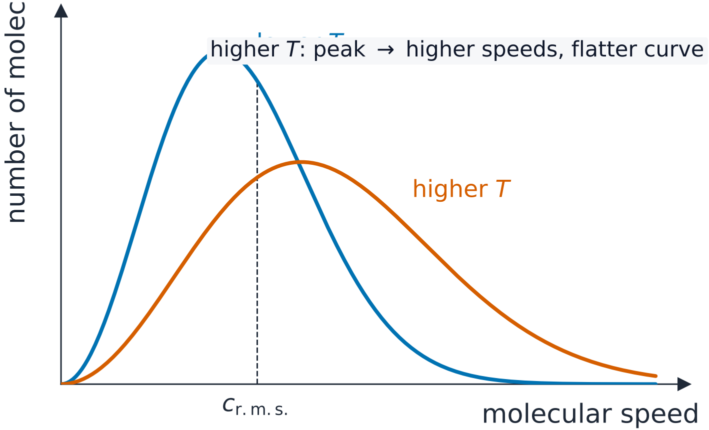
Method — Kinetic model

A gas is many identical molecules in **continuous random motion**; their collisions with the walls cause **pressure**: **Assumptions:** many molecules, random motion; their volume is negligible; collision time is negligible; no forces except in collisions; collisions are **elastic**; Newton's laws hold.



random motion — wall collisions cause pressure

Method — Kinetic model



The Maxwell-Boltzmann speed distribution

Method — Key results

$$pV = \tfrac{1}{3}Nm\langle c^2\rangle, \qquad c_{\text{r.m.s.}} = \sqrt{\langle c^2\rangle}.$$

Comparing with $pV = NkT$ gives the **mean translational KE** of a molecule:

$$\tfrac{1}{2}m\langle c^2\rangle = \tfrac{3}{2}kT.$$

So molecular KE depends only on **temperature**.

Practice 2.1 ▪ 2 marks

State **two** assumptions of the kinetic theory of gases.

Solution 2.1

Method: Kinetic model

Worked steps

Any two: many identical molecules in **random motion**; molecular **volume negligible**; **no forces** except during collisions; collisions are **elastic**; collision time negligible **B1** **B1**.

Practice 2.2 ▪ 2 marks

Explain how gas molecules cause **pressure** on the walls of a container.

Solution 2.2

Method: Kinetic model

Worked steps

Molecules collide with the walls and **change momentum** (rebound) **B1**; this exerts a **force** on the wall; force over the wall area is the **pressure** **B1**.

Practice 2.3 ▪ 1 mark

State what is meant by the **root-mean-square (r.m.s.) speed**.

Solution 2.3

Worked steps

The **square root of the mean of the squared speeds** of the molecules ($\sqrt{\langle c^2 \rangle}$) **B1**.

Practice 2.4 ▪ 1 mark

Write the relationship $pV = \frac{1}{3}Nm\langle c^2\rangle$ in words.

Solution 2.4

Method: Key results

Worked steps

The product of pressure and volume equals **one third** of (the number of molecules × molecular mass × mean-square speed) **B1**.

Exam-style 3.1 ▪ 1 mark

The **mean translational kinetic energy** of a gas molecule is:

A kT

B $\frac{3}{2}kT$

C $\frac{1}{2}mv^2$ only

D RT

Solution 3.1

Method: Key results

A kT

B $\frac{3}{2}kT$ ✓

C $\frac{1}{2}mv^2$ only

D RT

Worked steps

B — $\frac{3}{2}kT$.

Exam-style 3.2 ▪ 2 marks

Calculate the mean translational kinetic energy of a gas molecule at 300 K.
$(k = 1.38 \times 10^{-23}.)$

Solution 3.2

Method: Key results

Worked steps

$E = \frac{3}{2}kT = 1.5 \times (1.38 \times 10^{-23}) \times 300 = 6.2 \times 10^{-21}$ J **M1** **A1**.

Exam-style 3.3 ▪ 3 marks

A gas at 300 K has molecules of mass 5.3×10^{-26} kg. Calculate the **r.m.s. speed**.
($k = 1.38 \times 10^{-23}$.)

Solution 3.3

Method: Kinetic model

Worked steps

$\frac{1}{2}m\langle c^2\rangle = \frac{3}{2}kT \Rightarrow \langle c^2\rangle = \dfrac{3kT}{m}$ **M1**; $c_{\text{r.m.s.}} = \sqrt{\dfrac{3(1.38\times10^{-23})(300)}{5.3\times10^{-26}}}$ **M1** $= 4.8\times10^{2}\ \text{m s}^{-1}$ **A1**.

Exam-style 3.4 ▪ 2 marks

Explain, using the kinetic theory, why the pressure of a fixed mass of gas **increases** when it is heated at **constant volume**.

Solution 3.4

Method: Kinetic model

Worked steps

Heating raises the **mean KE / mean speed** of the molecules **B1**; they hit the walls **harder and more often**, so the pressure rises (volume fixed) **B1**.

Exam-style 3.5 ▪ 2 marks

A fixed sample of an ideal gas at thermodynamic temperature T has internal energy U.
(a) **State** what is meant by the **internal energy** of a system.
(b) **Explain** why the internal energy of an **ideal** gas is directly proportional to its thermodynamic temperature.
(c) The gas is first **compressed**, which raises its temperature to $3T$; work W is done on it during this compression. It is then **cooled at constant volume** back to $2T$. **Complete** the table, giving each entry in terms of some or all of W and U.

process	**work done on the gas**	**thermal energy supplied to the gas**	**increase in internal energy**
compression to $3T$	W	________	________
cooling to $2T$	________	________	________

Exam-style 3.5 ▪ 2 marks

(d) **Explain** why the work done on the gas during the second process is what it is.

Solution 3.5

Worked steps

(a) The **sum of the random kinetic and potential energies** of all the molecules **B1**, where "random" means the motion has no overall direction — the whole container moving does not count **B1**.

(b) In an ideal gas the molecules exert **no forces** on one another except during collisions, so the total **potential** energy is zero and the internal energy is entirely kinetic **B1**. The mean kinetic energy of a molecule is $\frac{3}{2}kT$, i.e. proportional to T, so the total is proportional to T as well **B1**.

(c) Internal energy is proportional to T, so at T it is U, at $3T$ it is $3U$ and at $2T$ it is $2U$.

Solution 3.5

process	**work done on the gas**	**thermal energy supplied**	**increase in internal energy**
compression to $3T$	W	$2U - W$	$+2U$
cooling to $2T$	0	$-U$	$-U$

B1 **B1** **B1** **B1** *for* $+2U$*; for* $-U$*; for the zero work; for both thermal-energy entries, from* $\Delta U = q + w$

(d) At **constant volume** the gas neither expands nor is compressed, so no work is done on or by it **B1**.