Exercise walk-through

Equation of state ▪ 15.2

eXercise

You must be able to

- use $pV = nRT$ and $pV = NkT$ (with T in kelvin)
- recall R, k and that $k = R/N_A$
- read a pV–T **graph**: gradient $= Nk$, so $N =$ gradient$/k$

Method — Equation of state

An **ideal gas** obeys $pV \propto T$. Two equal forms:

$$pV = nRT \quad (n = \text{moles}), \qquad pV = NkT \quad (N = \text{molecules}),$$

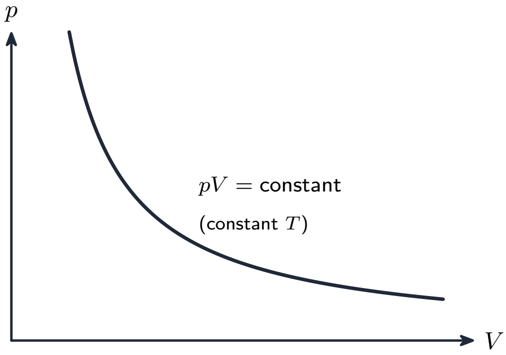
with $R = 8.31$ J mol^{-1} K^{-1}, $k = 1.38 \times 10^{-23}$ J K^{-1}, and $k = R/N_A$. **Always use Pa, m^3, K.**

Worked example. 0.50 mol at 300 K in 0.020 m^3:

$$p = \frac{nRT}{V} = \frac{0.50 \times 8.31 \times 300}{0.020} = 6.2 \times 10^4 \text{ Pa}.$$

Method — Boyle's law

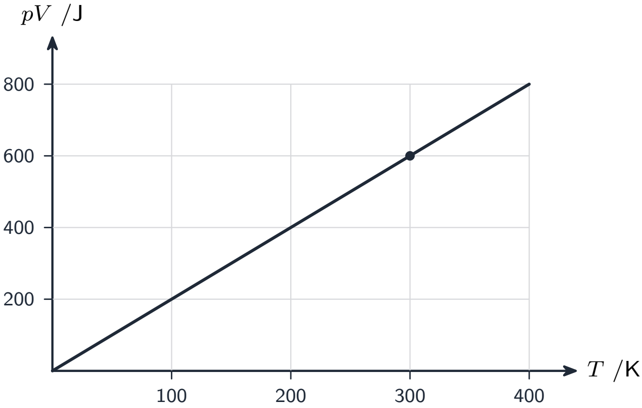
At constant T (and amount), $p \propto 1/V$, so $pV =$ constant:



Method — Reading a pV–T graph

For a fixed amount of gas, $pV = NkT$, so a graph of pV against T (in kelvin) is a **straight line through the origin** with **gradient** Nk:
A straight line **through the origin** confirms the gas is **ideal** ($pV \propto T$). To find the number of molecules, read the **gradient** and divide by k: $N = \dfrac{\text{gradient}}{k}$.
Worked example. The line above passes through $(300\ \text{K}, 600\ \text{J})$, so gradient $= 600/300 = 2.0\ \text{J K}^{-1} = Nk$; thus $N = \dfrac{2.0}{1.38 \times 10^{-23}} = 1.4 \times 10^{23}$.

Method — Reading a pV–T graph



Practice 2.1 ▪ 1 mark

State what is meant by an **ideal gas**.

Solution 2.1

Worked steps

A gas that obeys $pV \propto T$ (i.e. $pV = nRT$) exactly, at all p and T B1.

Practice 2.2 ▪ 2 marks

Write the **two** forms of the ideal-gas equation of state.

Solution 2.2

Method: Equation of state

Worked steps

$pV = nRT$ and $pV = NkT$ **B1** **B1**.

Practice 2.3 ▪ 1 mark

State the value of the **molar gas constant** R.

Solution 2.3

Worked steps

$R = 8.31$ J mol^{-1} K^{-1} **B1**.

Practice 2.4 ▪ 1 mark

A graph of pV against T for a fixed mass of ideal gas is a straight line through the origin. State what its **gradient** is equal to.

Solution 2.4

Method: Reading a \$pV\$–\$T\$ graph

Worked steps

Nk (the number of molecules $\times$ the Boltzmann constant) **B1**.

Exam-style 3.1 ▪ 1 mark

In $pV = nRT$, the temperature T must be in:

A °C

B K

C J

D mol

Solution 3.1

A °C

B K ✓

C J

D mol

Worked steps

B — kelvin.

Exam-style 3.2 ▪ 3 marks

0.50 mol of an ideal gas at 300 K occupies 0.020 m^3. Calculate the **pressure**.
($R = 8.31$.)

Solution 3.2

Worked steps

$$p = \frac{nRT}{V} = \frac{0.50 \times 8.31 \times 300}{0.020} \quad \text{M1}\,\text{M1} \quad = 6.2 \times 10^4\ \text{Pa} \quad \text{A1}.$$

Exam-style 3.3 ▪ 1 mark

For a fixed mass of gas, the graph of pV against T is a straight line through the origin passing through $(300\ \mathsf{K}, 600\ \mathsf{J})$.

(a) State what this tells you about the gas.

(b) Determine the number of molecules N in the sample. ($k = 1.38 \times 10^{-23}$.)

Solution 3.3

Method: Reading a \$pV\$–\$T\$ graph

Worked steps

(b) gradient $= \dfrac{600}{300} = 2.0$ J K^{-1} **M1**; this equals Nk **M1**; $N = \dfrac{2.0}{1.38 \times 10^{-23}} = 1.4 \times 10^{23}$ **A1**.

Exam-style 3.4 ▪ 1 mark

State **Boyle's law**.

Solution 3.4

Worked steps

At constant temperature (and amount of gas), the pressure is **inversely proportional** to the volume ($pV =$ constant) **B1**.

Exam-style 3.5 ▪ 2 marks

For an ideal gas, two equations are used together:

$$pV = NkT \qquad \text{and} \qquad pV = \frac{1}{3}Nm\langle c^2\rangle$$

(a) **State** the meaning of the symbol T in the first equation, and **identify** the constant k.
(b) **State** the meanings of the symbols m and $\langle c^2\rangle$ in the second equation.
(c) Use the two equations to **derive** an expression, in terms of k and T only, for the mean kinetic energy E_K of one molecule of the gas.
(d) Hence **state and explain** how the root-mean-square speed of the molecules changes when the thermodynamic temperature of the gas is **quadrupled**.

Solution 3.5

Worked steps

(a) T is the **thermodynamic (absolute) temperature**, measured in kelvin **B1**; k is the **Boltzmann constant** **B1**.

(b) m is the mass of **one molecule** **B1**; $\langle c^2 \rangle$ is the **mean square speed** — the average of the squares of the molecular speeds, not the square of the average **B1**.

(c) Setting the two right-hand sides equal: $NkT = \frac{1}{3}Nm\langle c^2 \rangle$ **M1**, so $\frac{1}{3}m\langle c^2 \rangle = kT$. The mean kinetic energy of one molecule is $E_K = \frac{1}{2}m\langle c^2 \rangle$ **M1**, which is $\frac{3}{2}$ times the previous line, so $E_K = \frac{3}{2}kT$ **A1**.

(d) $\frac{1}{2}m\langle c^2 \rangle \propto T$, so $\langle c^2 \rangle \propto T$ and $c_{\text{r.m.s.}} \propto \sqrt{T}$ **B1**; quadrupling T therefore **doubles** the r.m.s. speed **B1**.