Exercise walk-through

Gravitational potential ▪ 13.4

eXercise

You must be able to

- define **gravitational potential** and use $\phi = -\dfrac{GM}{r}$
- use $E_P = -\dfrac{GMm}{r}$ for two point masses
- explain why potential and E_P are negative

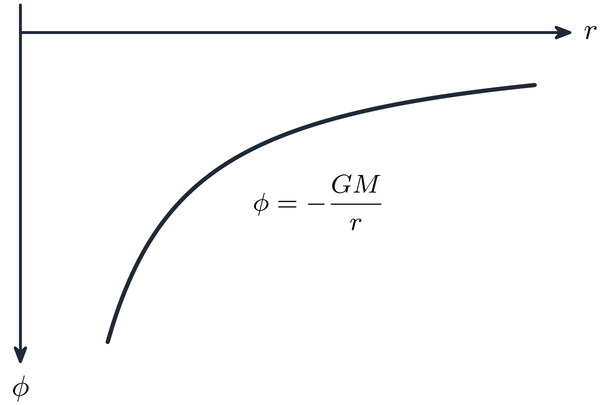
Method — Gravitational potential

Gravitational potential ϕ at a point is the **work done per unit mass** to bring a small test mass **from infinity** to that point. Potential is **zero at infinity** and **negative** everywhere else:

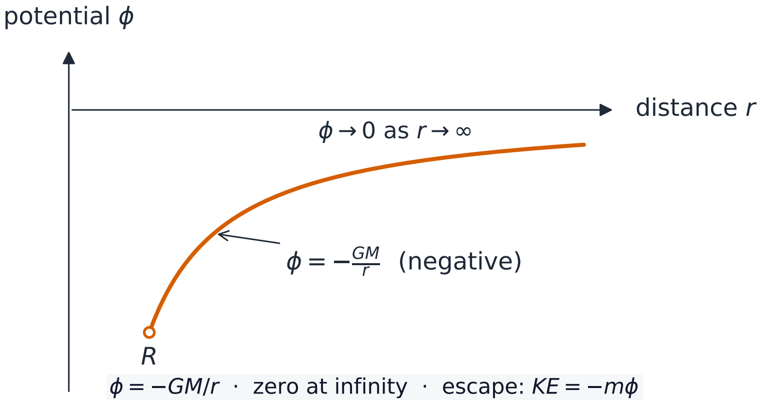
$$\phi = -\frac{GM}{r} \quad (\text{J kg}^{-1}), \qquad E_P = m\phi = -\frac{GMm}{r}.$$

For two radii, $\Delta E_P = GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$.

Method — Gravitational potential



Method — Gravitational potential



$\phi = -GM/r$ · zero at infinity · escape: $KE = -m\phi$

The gravitational potential well

Practice 2.1 ▪ 1 mark

Define **gravitational potential**.

Solution 2.1

Method: Gravitational potential

Worked steps

The **work done per unit mass** in bringing a small test mass from **infinity** to that point B1.

Practice 2.2 ▪ 1 mark

State **why** gravitational potential is **negative**.

Solution 2.2

Method: Gravitational potential

Worked steps

Gravity is **attractive**, so it does the work as the mass moves in from infinity (where $\phi = 0$); the potential therefore becomes **negative** B1.

Practice 2.3 ▪ 1 mark

State the unit of gravitational potential.

Solution 2.3

Method: Gravitational potential

Worked steps

$\mathrm{J\ kg^{-1}}$ **B1**.

Practice 2.4 ▪ 1 mark

Write the equation for the **gravitational potential energy** of two point masses.

Solution 2.4

Method: Gravitational potential

Worked steps

$$E_{\mathrm{P}} = -\frac{GMm}{r} \quad \boxed{\textbf{B1}}.$$

Exam-style 3.1 ▪ 1 mark

The gravitational potential at **infinity** is:

A a maximum positive value

B zero

C a large negative value

D infinite

Solution 3.1

Method: Gravitational potential

A a maximum positive value

B zero ✓

C a large negative value

D infinite

Worked steps

B — zero.

Exam-style 3.2 ▪ 3 marks

Calculate the gravitational potential at the Earth's surface. ($M = 6.0 \times 10^{24}$ kg, $R = 6.4 \times 10^{6}$ m, $G = 6.67 \times 10^{-11}$.)

Solution 3.2

Method: Gravitational potential

Worked steps

$$\phi = -\frac{GM}{R} = -\frac{(6.67 \times 10^{-11})(6.0 \times 10^{24})}{6.4 \times 10^{6}} \quad \textbf{M1} \textbf{M1} \quad = -6.3 \times 10^{7} \text{ J kg}^{-1} \quad \textbf{A1}.$$

Exam-style 3.3 ▪ 2 marks

Calculate the gravitational potential energy of a 1200 kg satellite at $r = 7.0 \times 10^6$ m from the Earth's centre.

Solution 3.3

Method: Gravitational potential

Worked steps

$$E_\mathsf{P} = -\frac{GMm}{r} = -\frac{(6.67 \times 10^{-11})(6.0 \times 10^{24})(1200)}{7.0 \times 10^{6}} \quad \textbf{M1} \quad = -6.9 \times 10^{10}\ \mathsf{J} \quad \textbf{A1}.$$

Exam-style 3.4 ▪ 2 marks

Explain why **two closer** masses have **more negative** gravitational potential energy.

Solution 3.4

Method: Gravitational potential

Worked steps

As r decreases, $-\dfrac{GMm}{r}$ becomes **more negative** B1; the masses are more **tightly bound** — more work would be needed to separate them to infinity B1.

Exam-style 3.5 ▪ 4 marks

A 1000 kg satellite orbiting a planet is moved from an orbit of radius 7.0×10^6 m to one of radius 1.4×10^7 m. Its gravitational potential energy **increases** by 2.86×10^{10} J. Calculate the **mass of the planet**, and give a **unit**. ($G = 6.67 \times 10^{-11}$.)

Solution 3.5

Method: Gravitational potential

Worked steps

$\Delta E_P = GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$ **M1**; $\frac{1}{r_1} - \frac{1}{r_2} = \frac{1}{7.0 \times 10^6} - \frac{1}{1.4 \times 10^7} = 7.14 \times 10^{-8}\ \text{m}^{-1}$ **M1**; $M = \frac{\Delta E_P}{Gm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)} = \frac{2.86 \times 10^{10}}{(6.67 \times 10^{-11})(1000)(7.14 \times 10^{-8})}$ **M1** $= 6.0 \times 10^{24}\ \text{kg}$ **A1**.