Past-paper walk-through

Gravitational field of a point mass ▪ 13.3

eXercise

Questions in this set

- 9702/41 N25 Q3 ▪ 9 marks
- 9702/43 J25 Q1 ▪ 9 marks
- 9702/44 J25 Q1 ▪ 11 marks

Reading the mark scheme

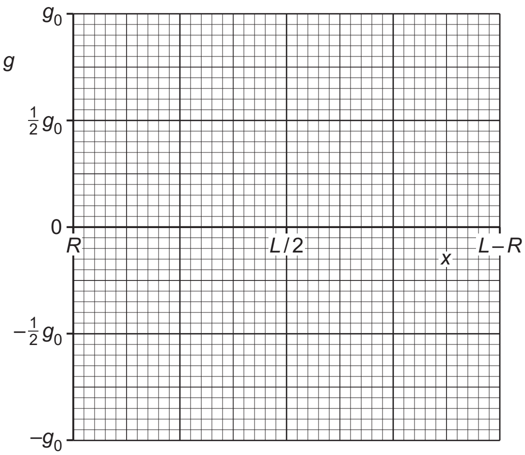
- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 3

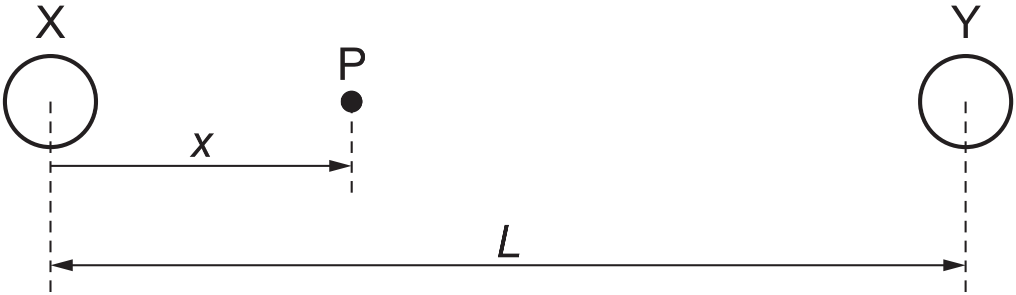
9702/41 N25 ▪ Q3 ▪ 9 marks

(a) Define gravitational field at a point. **[1]**

Question 3



Question 3



Question 3

9702/41 N25 ▪ Q3 ▪ 9 marks

(b)(i) By considering the force exerted by the point mass on a test mass of mass m placed at P, derive an equation for the gravitational field strength g at P, in terms of M and x. Identify any other symbols you use. **[2]**

(b)(ii) On Fig. 3.1, draw an arrow to indicate the direction of the gravitational field at P. **[1]**

(b)(iii) Point Q is at distance $\frac{x}{2}$ from the point mass, on the opposite side of the mass from P, as shown in Fig. 3.2.

Compare the gravitational field at Q with that at P. **[2]**

Question 3

9702/41 N25 ▪ Q3 ▪ 9 marks

(c) Two identical isolated uniform spheres X and Y each have radius R. The centres of the spheres are separated by distance L, as shown in Fig. 3.3.

Point P lies on the line joining the centres of X and Y, and is at a variable displacement x from the centre of sphere X. The gravitational field strength at the surface of each sphere is g_0.

On Fig. 3.4, sketch the variation with x of the gravitational field g at point P between $x = R$ and $x = L - R$. **[3]**

Solution 3

(a)

Mark scheme

- force per unit mass B1

Solution 3

(b)(i) Worked steps

"By considering the force on a test mass" fixes the route: start from Newton's law of gravitation and divide by the test mass.

- Force on the test mass at distance x: $F = \dfrac{GMm}{x^2}$.
- Gravitational field strength is force per unit mass: $g = \dfrac{F}{m}$.
- So $g = \dfrac{GMm}{x^2 m} = \dfrac{GM}{x^2}$, where G is the gravitational constant.

Solution 3

The mark for the derivation is the division by m — quoting the formula alone answers a different question.

Mark scheme

- F = GMm / x2 **C1**
- g = F / m g = [GMm / x2] / m = GM / x2 and G = gravitational constant **A1**

Solution 3

(b)(ii)

Worked steps

- A gravitational field is always attractive, so draw the arrow at P pointing **directly towards** the point mass, along the line joining them.

Mark scheme

- arrow drawn at P pointing directly towards the point mass B1

Solution 3

(b)(iii)

Mark scheme

- fields are in opposite directions **B1**
- field strength at Q is four times the field strength at P **B1**

Solution 3

(c) Worked steps

Between two equal masses the field points towards whichever sphere is nearer, so it reverses at the midpoint.

- The line starts at the surface of X, $(R, -g_0)$, and ends at the surface of Y, $(L - R, +g_0)$ — same size, opposite sign, because the spheres are identical.
- It crosses zero at the **midpoint**, $x = L/2$, where the two pulls cancel.
- The curve is shallow near the middle and steepens towards each surface, because each contribution goes as $1/x^2$.

Solution 3

Mark scheme

- line starting at (R, −g0) and ending at (L – R, +g0) **B1**
- line passing through (L / 2, 0) **B1**
- curve becoming shallower from R to (L / 2) and then steeper from (L / 2) to (L – R) **B1**

Question 1

9702/43 J25 ▪ Q1 ▪ 9 marks

1 **(a)** Define gravitational potential at a point.

Question 1

(b) Mars is a planet that may be considered to be an isolated uniform sphere of radius 3.4×10^6 m.

A satellite of mass 122 kg is in orbit around Mars at a constant height of 1.7×10^6 m above the surface of the planet.

The height of the orbit is increased to 6.8×10^6 m above the surface. This increases the gravitational potential energy of the satellite by 5.1×10^8 J.

(i) Show that the mass of Mars is 6.4×10^{23} kg.

Question 1

[3]

(ii) Calculate the gravitational potential ϕ at the surface of Mars. Give a unit with your answer.

Question 1

(c) The satellite in **(b)** is moved to an orbit in which the satellite remains at the same point above the surface of Mars.

(i) The orbit has a period of 25 hours.

State what can be deduced from this about the rotation of Mars on its axis.

Question 1

(ii) State **one** other feature of this orbit.

Question 1

[Total: 9]

Solution 1

(a)

Mark scheme

- work done per unit mass
- work (done in) moving mass from infinity (to the point)

Solution 1

(b)(i) Worked steps

The orbital radius is measured from the **centre**, so add the planet's radius first.

- $r = 3.4 \times 10^6 + 1.7 \times 10^6 = 5.1 \times 10^6$ m
- Gravity supplies the centripetal force: $\dfrac{GMm}{r^2} = mr\omega^2$, so $GM = r^3\omega^2$ with $\omega = \dfrac{2\pi}{T}$.
- $M = \dfrac{4\pi^2 r^3}{GT^2} = 6.4 \times 10^{23}$ kg, as required.

Solution 1

Mark scheme

- evidence of addition of 3.4×106 to 1.7×106 or 6.8×106
- $GM \times 122 / (5.1 \times 106)$ or $GM \times 122 / (10.2 \times 106)$
- $6.67 \times 10–11 \times M \times 122 \times [(5.1 \times 106)–1 – (10.2 \times 106)–1] = 5.1 \times 108$
- leading to $M = 6.4 \times 1023$ kg

Solution 1

(b)(ii) Worked steps

- $\phi = -\dfrac{GM}{r}$ at the **surface**, so $r = 3.4 \times 10^6$ m.
- $\phi = -\dfrac{6.67 \times 10^{-11} \times 6.4 \times 10^{23}}{3.4 \times 10^6}$
- $\phi = -1.3 \times 10^7$ J kg^{-1}

Solution 1

The unit is asked for, and the sign is part of the answer: potential is zero at infinity, so everywhere else it is negative.

Mark scheme

- $\phi = (–)\,(6.67 \times 10–11 \times 6.4 \times 1023)\ /\ (3.4 \times 106)$
- $= –1.3 \times 107$ J kg–1

Solution 1

(c)(i)

Mark scheme

- Mars takes (just under) 25 hours to rotate once on its axis

Solution 1

(c)(ii) Worked steps

A satellite that stays above one point on the surface has to match the planet's own rotation in **two** ways, and the question asks for the condition beyond the period.

- Its orbit must be **equatorial** — an orbit inclined to the equator carries the satellite north and south of its starting latitude, so it traces a figure of eight rather than staying put.
- It must travel **in the same direction as the planet's rotation**, west to east for Mars as for the Earth.

Solution 1

Either statement earns the mark. Both follow from the same requirement: the satellite's angular velocity must equal the planet's as a **vector**, not just in magnitude — same direction and same axis, and the period alone fixes only the magnitude.

Mark scheme

- orbit is equatorial
- or
- orbit is in same direction as direction of rotation of Mars
- 9702/43
- May/June 2025

Question 1

9702/44 J25 ▪ Q1 ▪ 11 marks

1 **(a)** Define gravitational field.

Question 1

(b) The gravitational field strength g at a distance x from the centre of a uniform spherical planet of mass M is given by the expression

$$g = \frac{GM}{x^2}$$

where G is the gravitational constant and distance x is greater than the radius of the planet.

(i) Describe the pattern of the field lines outside the planet that represent the gravitational field due to the planet.

Question 1

(ii) Explain why, for small changes in vertical height near the surface of the planet, g may be assumed to be constant.

Question 1

(c) Assume that the Earth is a uniform sphere. For the Earth, the product *GM* is equal to $3.99 \times 10^{14}\,\mathrm{m^3\,s^{-2}}$.

(i) Determine a value, to three significant figures, for the radius R of the Earth.

Question 1

(ii) Calculate the gravitational potential at the Earth’s surface. Give a unit with your answer.

Question 1

(d) Explain why the gravitational potential energy of two point masses is always negative.

Question 1

[Total: 11]

Solution 1

(a)

Mark scheme

- force per unit mass

Solution 1

(b)(i)

Mark scheme

- radial
- towards (centre of) planet

Solution 1

(b)(ii)

Mark scheme

- (changes in) height (very) much smaller than radius
- (radius + height)2 ≈ radius2
- or
- field lines are approximately parallel

Solution 1

(c)(i) Worked steps

At the surface the field strength and the mass product are linked by one equation, and GM is given.

- $g = \dfrac{GM}{R^2}$, so $gR^2 = GM$ and $R = \sqrt{\dfrac{GM}{g}}$
- $9.81 \times R^2 = 3.99 \times 10^{14}$
- $R = \sqrt{\dfrac{3.99 \times 10^{14}}{9.81}} = 6.38 \times 10^6$ m

Solution 1

The radius is **squared** in the field equation, so take the square root at the end. Forgetting it gives 4.07×10^{13}, which is larger than the Earth's orbit — the sense-check here is easy, because you already know the Earth's radius is a few thousand kilometres.

Mark scheme

- 9.81 × R2 = 3.99 × 1014
- R = 6.38 × 106 m

Solution 1

(c)(ii) Worked steps

Potential uses the **first** power of *r*, not the second, and it is negative.

- $\varphi = -\dfrac{GM}{R}$
- $\varphi = -\dfrac{3.99 \times 10^{14}}{6.38 \times 10^{6}}$
- $\varphi = -6.25 \times 10^{7}\ \mathrm{J\,kg^{-1}}$

Solution 1

Three things carry the marks: the **minus sign** (potential is zero at infinity and negative everywhere else, because gravity is attractive), the **single** power of *R*, and the unit $\mathrm{J\,kg^{-1}}$ — potential is energy per unit mass, not energy.
A quick way to check you have not confused the two formulae: $\varphi = -gR$ numerically, which is $-9.81 \times 6.38 \times 10^6 = -6.26 \times 10^7$, the same answer.

Mark scheme

- gravitational potential = – (GM / R)
- = – (3.99 × 1014) / (6.38 × 106)
- = – 6.25 × 107 J kg–1

Solution 1

(d) Worked steps

Two separate statements are wanted, and together they explain the minus sign in $\varphi = -\dfrac{GM}{r}$.

- Gravitational **potential is defined as zero at infinite separation** — that is the chosen reference point, not a physical property of infinity.
- Gravitational **force is always attractive**, so work must be done *against* the field to move a mass out to infinity. Bringing it in from there therefore leaves it with less energy than zero.

Solution 1

Put together: every point in a gravitational field is at a negative potential, and the closer you are to the mass the more negative it gets.
The contrast worth remembering is with electric potential, which uses the same zero at infinity but can be either sign, because the electric force can repel as well as attract.

Mark scheme

- potential (energy) zero at infinite separation
- (gravitational) force is attractive
- 9702/44
- May/June 2025