Exercise walk-through

Gravitational field ▪ 13.1

eXercise

You must be able to

- define **gravitational field strength** as force per unit mass
- represent a gravitational field with **field lines**

Method — Field strength

A **gravitational field** is a region where a mass feels a force. The **field strength** is the force per unit mass on a small test mass:

$$g = \frac{F}{m} \quad (\mathrm{N\ kg^{-1}}).$$

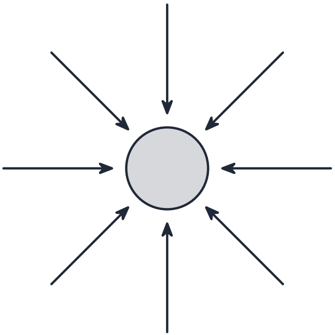
It is a **vector**, pointing towards the source mass (the same value as the free-fall acceleration).

Method — Field lines

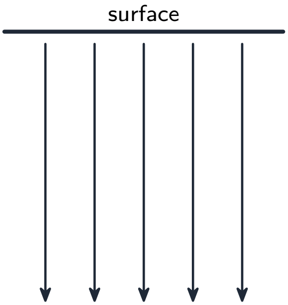
- around a **point mass / sphere**: **radial**, pointing **inwards**;
- near a surface (small region): **uniform** — parallel, equally spaced, pointing down.

Closer lines → stronger field.

Method — Field lines

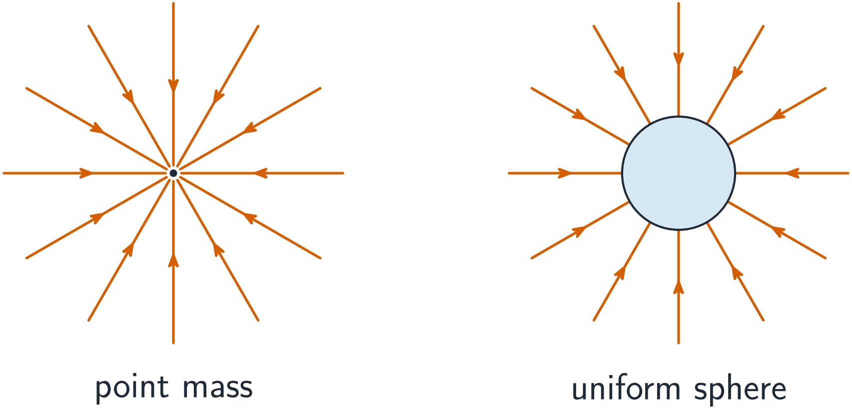


(a) radial field



(b) uniform field

Method — Field lines



The radial gravitational field of a sphere

Practice 2.1 ▪ 1 mark

Define **gravitational field strength**.

Solution 2.1

Worked steps

The **gravitational force per unit mass** on a small test mass at that point ($g = F/m$) B1.

Practice 2.2 ▪ 1 mark

State the unit of gravitational field strength.

Solution 2.2

Worked steps

N kg^{-1} **B1**.

Practice 2.3 ▪ 1 mark

State what a gravitational **field line** represents.

Solution 2.3

Method: Field lines

Worked steps

The **direction of the gravitational force** on a (test) mass at that point B1.

Practice 2.4 ▪ 1 mark

State the shape of the gravitational field around a **point mass**.

Solution 2.4

Method: Field lines

Worked steps

Radial, pointing **inwards** towards the mass B1.

Exam-style 3.1 ▪ 1 mark

Gravitational field strength is equal to:

A force $\times$ mass

B force $\div$ mass

C mass $\div$ force

D mass $\times$ acceleration

Solution 3.1

A force $\times$ mass

B force $\div$ mass ✓

C mass $\div$ force

D mass $\times$ acceleration

Worked steps

B — force $\div$ mass.

Exam-style 3.2 ▪ 2 marks

A mass of 2.0 kg experiences a gravitational force of 19.6 N. Find the **gravitational field strength**.

Solution 3.2

Method: Field strength

Worked steps

$g = F/m = 19.6/2.0 = 9.8$ N kg^{-1} **M1** **A1**.

Exam-style 3.3 ▪ 1 mark

Explain what **closer** field lines indicate.

Solution 3.3

Method: Field lines

Worked steps

A **stronger** gravitational field B1.

Exam-style 3.4 ▪ 2 marks

Sketch the gravitational field lines (a) around a point mass and (b) near a flat surface.

Solution 3.4

Method: Field lines

Worked steps

(a) **radial** lines pointing inward to the mass B1; (b) **parallel, equally spaced** lines pointing down towards the surface B1.

Exam-style 3.5 ▪ 1 mark

A point mass M sits at the origin. Point P is a distance x from it.

(a) **Define** the **gravitational field** at a point.

(b) By considering the force on a test mass m placed at P, **derive** an expression for the gravitational field strength g at P in terms of M and x. State the meaning of any other symbol you use.

(c) Point Q lies on the **opposite** side of the mass from P, at a distance $\dfrac{x}{2}$ from it. **Compare** the gravitational field at Q with the field at P.

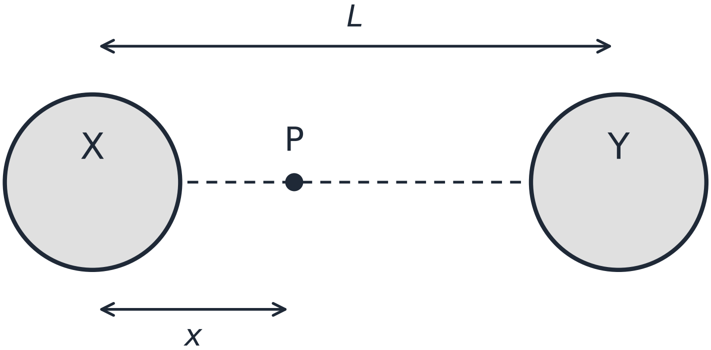
(d) Two identical uniform spheres X and Y, each of radius R, have their centres a distance L apart. Point P now lies on the line joining their centres, a displacement x from the centre of X. The gravitational field strength at the surface of each sphere is g_0.

On the axes below, **sketch** how the field g at P varies with x, from $x = R$ to $x = L - R$. Take the direction from X towards Y as positive.

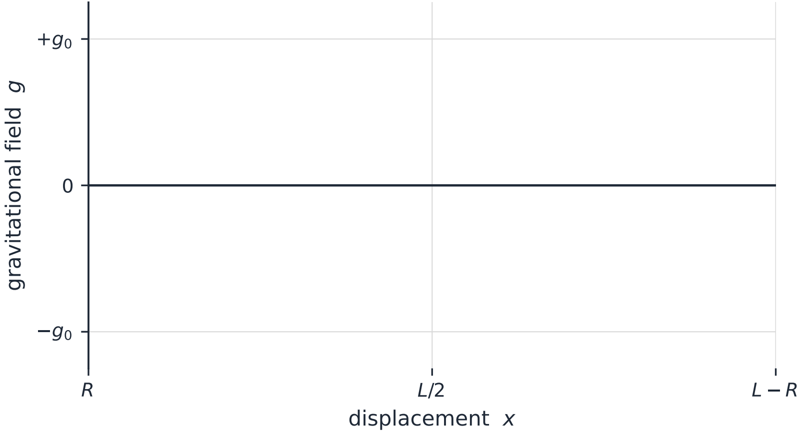
Exam-style 3.5 ▪ 1 mark

(e) **Explain** why the field is zero at the midpoint, even though neither sphere has been removed.

Exam-style 3.5 ▪ 1 mark



Exam-style 3.5 ▪ 1 mark



Solution 3.5

Method: Field lines

Worked steps

(a) The gravitational **force per unit mass** acting on a small test mass placed at that point **B1**.

(b) The force on the test mass is $F = \dfrac{GMm}{x^2}$, where G is the gravitational constant **M1**; the field strength is the force per unit mass, so $g = \dfrac{F}{m} = \dfrac{GM}{x^2}$ **A1**.

(c) The fields point in **opposite directions**, since each points towards the mass **B1**; and because $g \propto \dfrac{1}{x^2}$, halving the distance makes the field at Q **four times** the field at P **B1**.

Solution 3.5

(d) The curve starts at $(R,\ -g_0)$, since at X's surface the field points back towards X, i.e. negative **B1**; it passes through zero at the midpoint $x = L/2$ **B1**; and it ends at $(L - R,\ +g_0)$, becoming shallower up to the midpoint and steeper after it **B1**.
(e) The two spheres are identical and the midpoint is equidistant from both, so their fields there are **equal in magnitude** **B1** and **opposite in direction**, and since field strength is a vector the two cancel to zero **B1**. (The gravitational *potential* there is not zero — only the field is.)