

Past-paper walk-through

Centripetal acceleration ■ 12.2

eXercise

Questions in this set

- 9702/41 N25 Q1 ■ 15 marks
- 9702/42 N25 Q1 ■ 9 marks
- 9702/42 M25 Q1 ■ 9 marks

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- **//** separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = “or words to that effect” – the wording need not match.

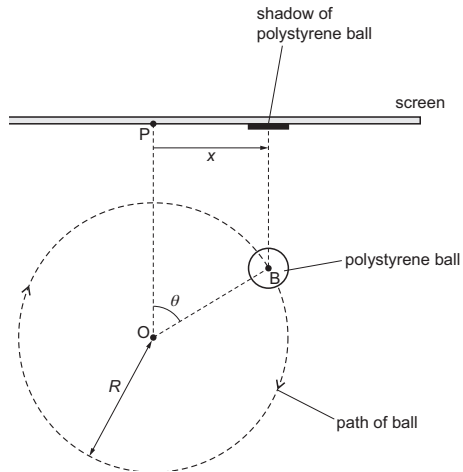
Question 1

9702/41 N25 ■ Q1 ■ 15 marks

(a) In terms of velocity and acceleration, describe uniform circular motion of an object.

[2]

Question 1



Question 1



Question 1

9702/41 N25 ■ Q1 ■ 15 marks

- (b)(i)** State an expression, in terms of R and ω , for the speed v of the ball. [1]
- (b)(ii)** Determine an expression, in terms of v and ω , for the centripetal acceleration of the ball. [2]

Question 1

9702/41 N25 ■ Q1 ■ 15 marks

(c)(i) Determine an expression, in terms of R and θ , for the displacement x of the shadow from P. [1]

(c)(ii) The value of θ is zero at time $t = 0$.
State an expression for θ in terms of ω and t . [1]

(c)(iii) Use your answers in **(c)(i)** and **(c)(ii)** to show that x is given by

$$x = R \sin \omega t$$

(c)(iv) Explain, with reference to the equation in **(c)(iii)**, why the motion of the shadow of the ball on the screen may be modelled as simple harmonic. [1]

Question 1

9702/41 N25 ■ Q1 ■ 15 marks

(d)(i) the amplitude

[1]

(d)(ii) the period

[2]

(d)(iii) the maximum acceleration.

[2]

Question 1

9702/41 N25 ■ Q1 ■ 15 marks

(e) On Fig. 1.1, draw, and label with the letter A, the position of the shadow on the screen when the shadow has its maximum positive acceleration. **[1]**

Solution 1

(a)

Mark scheme

- velocity and acceleration both have constant magnitude (B1)
- velocity is (always) perpendicular to acceleration (B1)

Solution 1

(b)(i)

Mark scheme

■ $v = R\omega$ **A1**

Solution 1

(b)(ii) Worked steps

Two standard forms are available, and the question wants the one in v and ω .

- $a = R\omega^2$, and $v = R\omega$, so $R = \frac{v}{\omega}$.

- Substituting: $a = \frac{v}{\omega}\omega^2$

- $a = v\omega$

Solution 1

Mark scheme

- $a = R\omega^2$ or $a = v^2 / R$ (C1)
- $a = v\omega$ (A1)

Solution 1

(c)(i)

Worked steps

The shadow's displacement is the component of the radius along the screen.

- With θ measured from the line through P, the perpendicular component is $x = R \sin \theta$.

Mark scheme

- $x = R \sin \theta$ (A1)

Solution 1

(c)(ii)

Mark scheme

■ $\theta = \omega t$ **A1**

Solution 1

(c)(iii)

Worked steps

- The ball turns at constant angular speed, so the angle it has swept is $\theta = \omega t$.
- Substituting that into $x = R \sin \theta$ gives $x = R \sin \omega t$, as required.

This is the whole reason a shadow of uniform circular motion is simple harmonic: the displacement is a sine of a quantity that grows linearly with time.

Mark scheme

- clear substitution of $\theta = \omega t$ into $x = R \sin \theta$ leading to $x = R \sin \omega t$ **A1**

Solution 1

(c)(iv)

Mark scheme

- equation is of the form $x = x_0 \sin \omega t$ (so simple harmonic motion) **B1**

Solution 1

(d)(i)

Worked steps

The shadow sweeps the full diameter of the circle, so its amplitude is the **radius**, not the diameter.

$$\blacksquare \text{ amplitude} = \frac{0.46}{2} = 0.23 \text{ m}$$

Mark scheme

- amplitude = $0.46 / 2$
- = 0.23 m (A1)

Solution 1

(d)(ii)

Worked steps

- $\omega = \frac{2\pi}{T}$, so $T = \frac{2\pi}{\omega} = \frac{2\pi}{1.9}$
- $T = 3.3 \text{ s}$

Mark scheme

- $\omega = 2\pi / T$ (C1)
- period = $2\pi / 1.9$
- = 3.3 s (A1)

Solution 1

(d)(iii)

Worked steps

- For SHM the maximum acceleration is $a_0 = \omega^2 x_0$.
- $a_0 = 1.9^2 \times 0.23$
- $a_0 = 0.83 \text{ m s}^{-2}$

Note it is the **amplitude** from (d)(i) that goes in here, not the diameter — the commonest slip on this question.

Mark scheme

- $a_0 = \omega^2 x_0$ (C1)
- $= 1.9^2 \times 0.23$
- $= 0.83 \text{ m s}^{-2}$ (A1)

Solution 1

(e) Worked steps

The shadow is the projection of the ball onto the screen, so it sits directly below (or beside) wherever the ball is on its circle.

- The shadow is furthest from the centre when the ball is at the **edge** of its circular path as seen from the screen.
- Mark A above the left-hand edge of the circular path — the point where the ball's displacement across the screen is greatest.

Solution 1

That is also where the shadow is momentarily at rest, which is what makes the motion simple harmonic: maximum displacement, zero velocity.

Mark scheme

- shadow on screen, labelled A, above left-hand edge of the circular path (B1)

Question 1

9702/42 N25 ■ Q1 ■ 9 marks

The Earth may be considered as a uniform sphere of radius 6.37×10^6 m.

(a)(i) Show that the radius of the circle around which Cambridge moves is 3.90×10^6 m. [1]

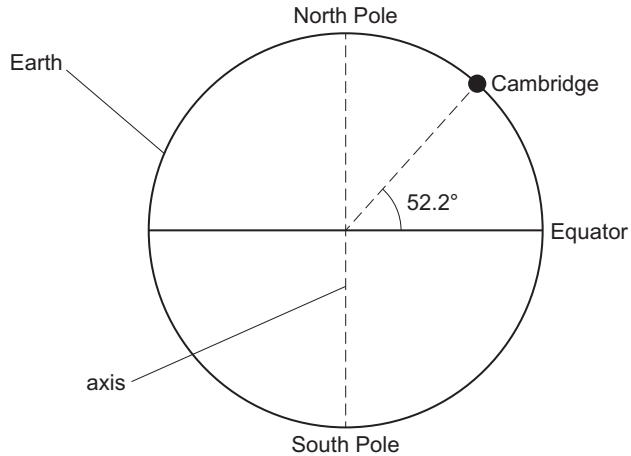
(a)(ii) Calculate the speed at which Cambridge moves around the circle. [3]

(b)(i) Determine the magnitude of the resultant force that acts to cause the circular motion of the student. [2]

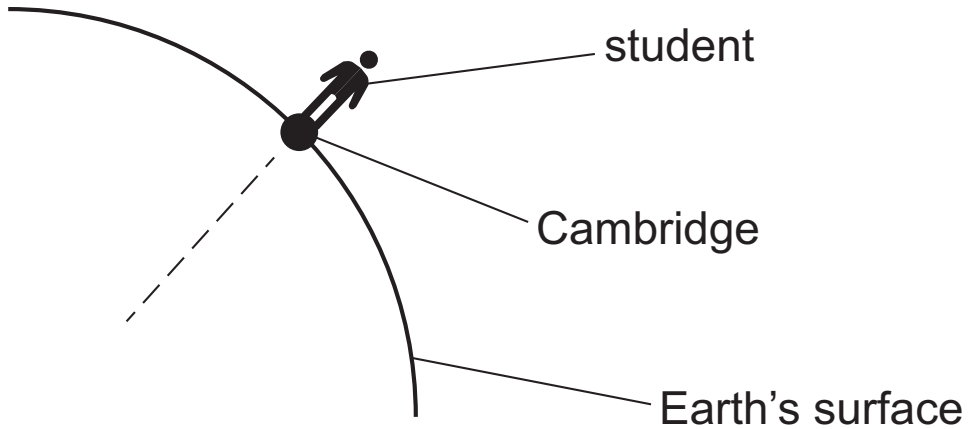
(b)(ii) On Fig. 1.2, draw an arrow to show the direction of the resultant force that acts on the student. [1]

(b)(iii) On Fig. 1.3, draw labelled arrows from the student to show the directions of the forces that act on the student to cause the resultant force in **(b)(ii)**. [2]

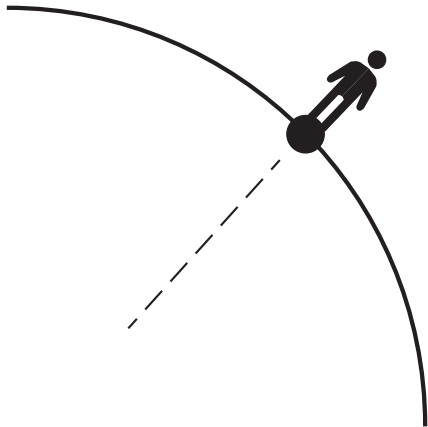
Question 1



Question 1



Question 1



Solution 1

(a)(i)

Worked steps

Cambridge does not go round the Earth's full radius: it traces a circle of latitude, whose radius is the perpendicular distance to the axis.

- $r = R \cos(\text{latitude}) = 6.37 \times 10^6 \times \cos 52.2^\circ$
- $r = 3.90 \times 10^6 \text{ m}$, as required.

Mark scheme

- radius = $6.37 \times 10^6 \times \cos 52.2^\circ = 3.90 \times 10^6 \text{ m}$ **A1**

Solution 1

(a)(ii) Worked steps

■ One rotation takes a day: $T = 24 \times 60 \times 60 = 86\,400 \text{ s}$.

$$\text{■ } v = \frac{2\pi r}{T} = \frac{2\pi \times 3.90 \times 10^6}{86\,400}$$

$$\text{■ } v = 280 \text{ m s}^{-1}$$

Solution 1

Mark scheme

- period = 24 hours (C1)
- $v = 2\pi r / T$ or $v = r\omega$ and $\omega = 2\pi / T$ (C1)
- $v = (2\pi \times 3.90 \times 10^6) / (24 \times 60 \times 60)$
- = 280 m s⁻¹ (A1)

Solution 1

(b)(i) Worked steps

Use the radius of the circle actually travelled — the one from part (a)(i), not the Earth's radius.

$$\blacksquare F = \frac{mv^2}{r} = \frac{58.6 \times 280^2}{3.90 \times 10^6}$$

$$\blacksquare F = 1.2 \text{ N}$$

Solution 1

Worth noticing: that is the **resultant** force needed to keep the student on the circle, and it is tiny beside their weight of about 575 N — which is why the rotation of the Earth makes so little difference to what a set of scales reads.

Mark scheme

- $F = mv^2 / r$ (C1)
- $= (58.6 \times 2802) / (3.90 \times 10^6)$
- $= 1.2 \text{ N}$ (A1)

Solution 1

(b)(ii)

Worked steps

The resultant force points to the **centre of the circle** the student travels — that circle of latitude, whose centre is on the Earth's axis, not at the Earth's centre.

- Draw the arrow horizontally, towards the axis (to the left on Fig. 1.2).

Mark scheme

- arrow pointing horizontally to the left (B1)

Solution 1

(b)(iii) Worked steps

Two forces act, and they do not line up — that is the point of the question.

- **Weight**, drawn from the student towards the Earth's centre, along the dotted line, and labelled.
- **Normal contact force** from the ground, drawn from the student slightly off that line: its direction is such that the two together leave a small resultant pointing at the axis.

Solution 1

The contact force is therefore not exactly opposite the weight, which is why a plumb line does not point exactly at the Earth's centre.

Mark scheme

- arrow from student pointing along the dotted line, labelled 'weight' (B1)
- upwards arrow from student pointing in a direction to the left of normal and above the tangent to the Earth, labelled 'contact force' (B1)

Question 1

9702/42 M25 ■ Q1 ■ 9 marks

- 1 A steel ball is placed on the inside surface of a hollow circular cone. The ball moves in a horizontal circle at constant speed, as shown in Fig. 1.1.

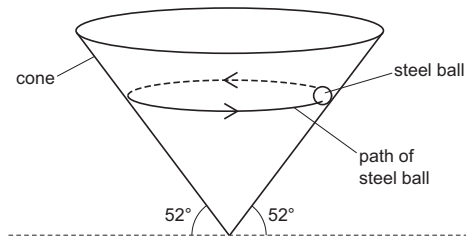


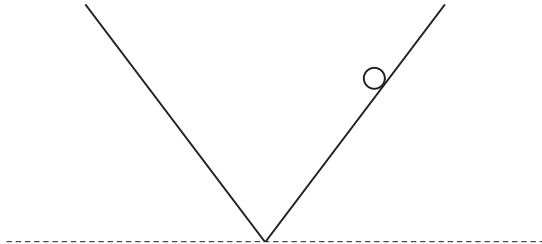
Fig. 1.1

The angle of the side of the cone to the horizontal is 52° . There is no friction between the ball and

Question 1

the cone.

(a) Fig. 1.2 shows a cross-section through the cone and the steel ball.



Question 1

Fig. 1.2

On Fig. 1.2, draw labelled arrows to show the **two** forces acting on the ball. [1]

(b) Describe how the forces acting on the ball cause its acceleration to be centripetal.

Question 1

(c) The ball moves in a circle of radius 0.15 m.

Show that the speed of the ball is 1.4 ms^{-1} .

Question 1

[3]

- (d) Calculate the angular speed ω of the ball.

Question 1

(e) The speed of the ball is increased.

Explain why the radius of the circular path of the ball increases.

Question 1

[Total: 9]

Solution 1

(a)

Mark scheme

- arrow vertically downwards labelled 'weight' and arrow perpendicular to cone, inwards and upwards, labelled 'normal
- contact force'

Solution 1

(b)

Mark scheme

- vertical component of contact force = weight (so no resultant force vertically)
- horizontal component of contact force is resultant force towards centre (of circle)

Solution 1

(c) Worked steps

Two forces act: the weight and the normal contact force from the cone. Their resultant is horizontal and provides the centripetal force.

- Resolving, the resultant acceleration is $a = g \tan 52^\circ$.
- For circular motion, $a = \frac{v^2}{r}$.
- So $\frac{v^2}{0.15} = 9.81 \tan 52^\circ$
- $v^2 = 0.15 \times 12.6 = 1.88$, giving $v = 1.4 \text{ m s}^{-1}$, as required.

Solution 1

Mark scheme

- $a = v^2 / r$
- $a = g \tan 52^\circ$
- $9.81 \times \tan 52^\circ = v^2 / 0.15$ leading to $v = 1.4 \text{ m s}^{-1}$

Solution 1

(d)

Worked steps

- $v = r\omega$, so $\omega = \frac{v}{r} = \frac{1.4}{0.15}$
- $\omega = 9.3 \text{ rad s}^{-1}$

Mark scheme

- $v = r\omega$ or $a = r\omega^2$
- $\omega = 1.4 / 0.15$ or $\sqrt{(9.81 \times \tan 52^\circ / 0.15)}$
- $= 9.3 \text{ rad s}^{-1}$

Solution 1

(e)

Worked steps

- The cone's angle is fixed, so the acceleration $g \tan 52^\circ$ is the **same** whatever the speed.
- With $a = \frac{v^2}{r}$ constant, v^2 is proportional to r .
- So a faster ball must move in a **larger** circle — it rides higher up the cone.

Mark scheme

- same resultant force / same acceleration so v^2 is proportional to r (so if speed increases radius must also increase)