Past-paper walk-through

Kinematics of uniform circular motion ▪ 12.1

eXercise

Questions in this set

- 9702/41 N25 Q1 ▪ 15 marks
- 9702/42 N25 Q1 ▪ 9 marks
- 9702/42 J25 Q1 ▪ 10 marks
- 9702/42 M25 Q1 ▪ 9 marks

Reading the mark scheme

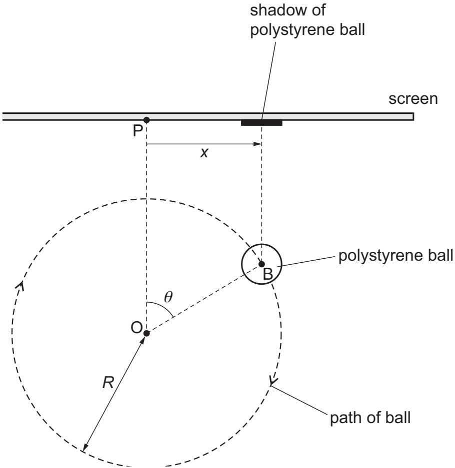
- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 1

9702/41 N25 ▪ Q1 ▪ 15 marks

(a) In terms of velocity and acceleration, describe uniform circular motion of an object. **[2]**

Question 1



shadow of
polystyrene ball
screen
P
x
polystyrene ball
B
θ
O
R
path of ball

Question 1

Question 1

9702/41 N25 ▪ Q1 ▪ 15 marks

(b)(i) State an expression, in terms of R and ω, for the speed v of the ball. **[1]**

(b)(ii) Determine an expression, in terms of v and ω, for the centripetal acceleration of the ball. **[2]**

Question 1

9702/41 N25 ▪ Q1 ▪ 15 marks

(c)(i) Determine an expression, in terms of R and θ, for the displacement x of the shadow from P. **[1]**

(c)(ii) The value of θ is zero at time $t = 0$.
State an expression for θ in terms of ω and t. **[1]**

(c)(iii) Use your answers in **(c)(i)** and **(c)(ii)** to show that x is given by

$$x = R \sin \omega t$$

[1]

(c)(iv) Explain, with reference to the equation in **(c)(iii)**, why the motion of the shadow of the ball on the screen may be modelled as simple harmonic. **[1]**

Question 1

9702/41 N25 ▪ Q1 ▪ 15 marks

(d)(i) the amplitude **[1]**

(d)(ii) the period **[2]**

(d)(iii) the maximum acceleration. **[2]**

Question 1

9702/41 N25 ▪ Q1 ▪ 15 marks

(e) On Fig. 1.1, draw, and label with the letter A, the position of the shadow on the screen when the shadow has its maximum positive acceleration. **[1]**

Solution 1

(a)

Mark scheme

- velocity and acceleration both have constant magnitude **B1**
- velocity is (always) perpendicular to acceleration **B1**

Solution 1

(b)(i)

Mark scheme

- $v = R\omega$ **A1**

Solution 1

(b)(ii) Worked steps

Two standard forms are available, and the question wants the one in v and ω.

- $a = R\omega^2$, and $v = R\omega$, so $R = \dfrac{v}{\omega}$.
- Substituting: $a = \dfrac{v}{\omega}\omega^2$
- $a = v\omega$

Solution 1

Mark scheme

- $a = R\omega 2$ or $a = v2 / R$ **C1**
- $a = v\omega$ **A1**

Solution 1

(c)(i)

Worked steps

The shadow's displacement is the component of the radius along the screen.

- With θ measured from the line through P, the perpendicular component is $x = R\sin\theta$.

Mark scheme

- x = R sin θ **A1**

Solution 1

(c)(ii)

Mark scheme

- $\theta = \omega \mathrm{t}$ **A1**

Solution 1

(c)(iii)

Worked steps

- The ball turns at constant angular speed, so the angle it has swept is $\theta = \omega t$.
- Substituting that into $x = R \sin\theta$ gives $x = R \sin\omega t$, as required.

This is the whole reason a shadow of uniform circular motion is simple harmonic: the displacement is a sine of a quantity that grows linearly with time.

Mark scheme

- clear substitution of $\theta = \omega$t into x = R sin θ leading to x = R sin ωt **A1**

Solution 1

(c)(iv)

Mark scheme

- equation is of the form $x = x0 \sin \omega t$ (so simple harmonic motion) **B1**

Solution 1

(d)(i)

Worked steps

The shadow sweeps the full diameter of the circle, so its amplitude is the **radius**, not the diameter.

- amplitude $= \dfrac{0.46}{2} = 0.23$ m

Mark scheme

- amplitude $=$ 0.46 / 2
- $=$ 0.23 m **A1**

Solution 1

(d)(ii)

Worked steps

- $\omega = \dfrac{2\pi}{T}$, so $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{1.9}$
- $T = 3.3$ s

Mark scheme

- $\omega = 2\pi$ / T **C1**
- period $= 2\pi$ / 1.9
- $=$ 3.3 s **A1**

Solution 1

(d)(iii)

Worked steps

- For SHM the maximum acceleration is $a_0 = \omega^2 x_0$.
- $a_0 = 1.9^2 \times 0.23$
- $a_0 = 0.83$ m s^{-2}

Note it is the **amplitude** from (d)(i) that goes in here, not the diameter — the commonest slip on this question.

Mark scheme

- a0 $=$ ω2x0 **C1**
- $=$ 1.92 $\times$ 0.23
- $=$ 0.83 m s–2 **A1**

Solution 1

(e) Worked steps

The shadow is the projection of the ball onto the screen, so it sits directly below (or beside) wherever the ball is on its circle.

- The shadow is furthest from the centre when the ball is at the **edge** of its circular path as seen from the screen.
- Mark A above the left-hand edge of the circular path — the point where the ball's displacement across the screen is greatest.

Solution 1

That is also where the shadow is momentarily at rest, which is what makes the motion simple harmonic: maximum displacement, zero velocity.

Mark scheme

- shadow on screen, labelled A, above left-hand edge of the circular path **B1**

Question 1

9702/42 N25 ▪ Q1 ▪ 9 marks

The Earth may be considered as a uniform sphere of radius 6.37×10^6 m.

(a)(i) Show that the radius of the circle around which Cambridge moves is 3.90×10^6 m. **[1]**

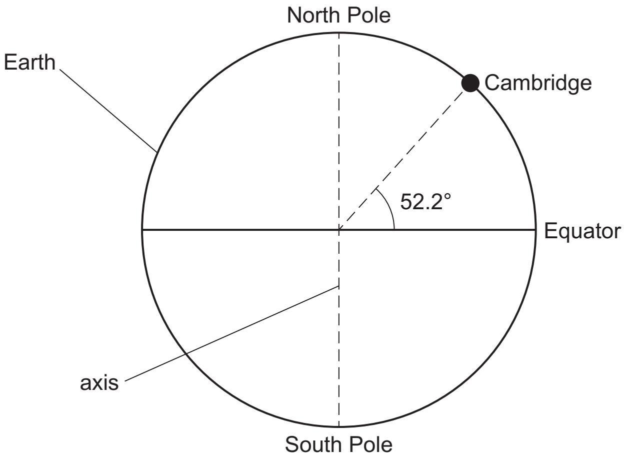
(a)(ii) Calculate the speed at which Cambridge moves around the circle. **[3]**

(b)(i) Determine the magnitude of the resultant force that acts to cause the circular motion of the student. **[2]**

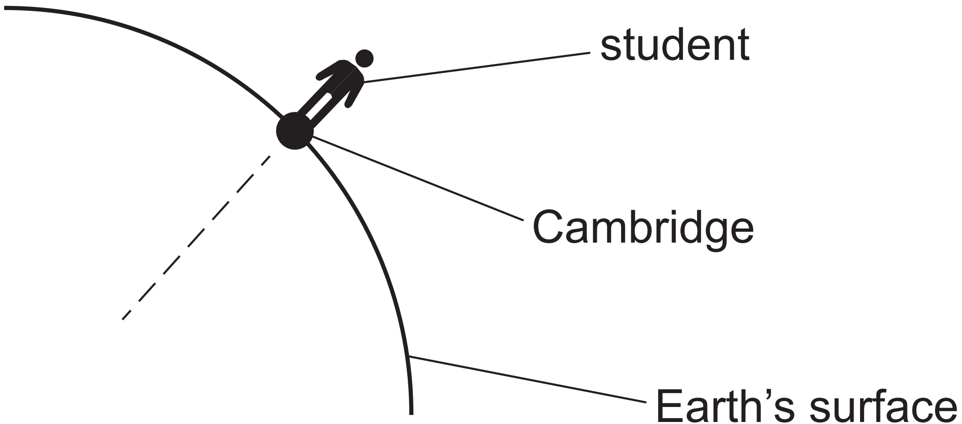
(b)(ii) On Fig. 1.2, draw an arrow to show the direction of the resultant force that acts on the student. **[1]**

(b)(iii) On Fig. 1.3, draw labelled arrows from the student to show the directions of the forces that act on the student to cause the resultant force in **(b)(ii)**. **[2]**

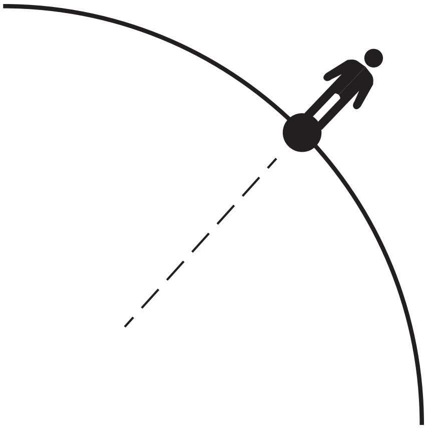
Question 1



Question 1



Question 1



Solution 1

(a)(i)

Worked steps

Cambridge does not go round the Earth's full radius: it traces a circle of latitude, whose radius is the perpendicular distance to the axis.

- $r = R\cos(\text{latitude}) = 6.37 \times 10^6 \times \cos 52.2°$
- $r = 3.90 \times 10^6$ m, as required.

Mark scheme

- radius = 6.37 × 106 × cos 52.2° = 3.90 × 106 m **A1**

Solution 1

(a)(ii) Worked steps

- One rotation takes a day: $T = 24 \times 60 \times 60 = 86\,400$ s.
- $v = \dfrac{2\pi r}{T} = \dfrac{2\pi \times 3.90 \times 10^6}{86\,400}$
- $v = 280$ m s^{-1}

Solution 1

Mark scheme

- period = 24 hours C1
- $v = 2\pi r$ / T or $v = r\omega$ and $\omega = 2\pi$ / T C1
- $v = (2\pi \times 3.90 \times 106) / (24 \times 60 \times 60)$
- = 280 m s–1 A1

Solution 1

(b)(i) Worked steps

Use the radius of the circle actually travelled — the one from part (a)(i), not the Earth's radius.

- $F = \dfrac{mv^2}{r} = \dfrac{58.6 \times 280^2}{3.90 \times 10^6}$
- $F = 1.2$ N

Solution 1

Worth noticing: that is the **resultant** force needed to keep the student on the circle, and it is tiny beside their weight of about 575 N — which is why the rotation of the Earth makes so little difference to what a set of scales reads.

Mark scheme

- $F = mv2 / r$ **C1**
- $= (58.6 \times 2802) / (3.90 \times 106)$
- $= 1.2$ N **A1**

Solution 1

(b)(ii)

Worked steps

The resultant force points to the **centre of the circle** the student travels — that circle of latitude, whose centre is on the Earth's axis, not at the Earth's centre.

- Draw the arrow horizontally, towards the axis (to the left on Fig. 1.2).

Mark scheme

- arrow pointing horizontally to the left **B1**

Solution 1

(b)(iii) Worked steps

Two forces act, and they do not line up — that is the point of the question.

- **Weight**, drawn from the student towards the Earth's centre, along the dotted line, and labelled.
- **Normal contact force** from the ground, drawn from the student slightly off that line: its direction is such that the two together leave a small resultant pointing at the axis.

Solution 1

The contact force is therefore not exactly opposite the weight, which is why a plumb line does not point exactly at the Earth's centre.

Mark scheme

- arrow from student pointing along the dotted line, labelled 'weight' **B1**
- upwards arrow from student pointing in a direction to the left of normal and above the tangent to the Earth, labelled 'contact force' **B1**

Question 1

9702/42 J25 ▪ Q1 ▪ 10 marks

(a) Define the radian. **[1]**

(b)(i) Calculate the angular speed of the rear wheel. **[2]**

(b)(ii) Calculate the period of rotation of the small cog. **[2]**

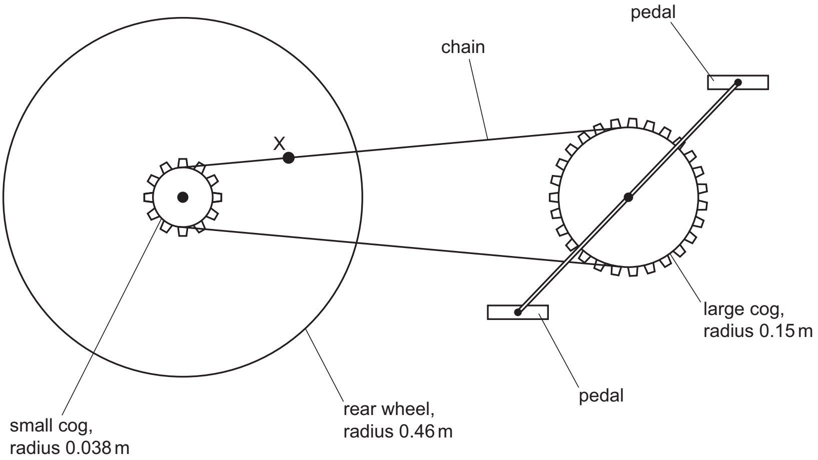
(b)(iii) Show that the distance moved by point X on the chain during one full rotation of the small cog is 0.24 m. **[1]**

(b)(iv) Use the information in **(b)(iii)** to determine the angle through which the large cog rotates during one full rotation of the small cog. **[2]**

(c) The chain of the bicycle in **(b)** is moved onto a smaller cog fixed to the rear wheel. The speed of the bicycle does not change.

Explain, without calculation, the effect of this change on the angular speed of the pedals. **[2]**

Question 1



pedal
chain
X
small cog,
radius 0.038 m
rear wheel,
radius 0.46 m
pedal
large cog,
radius 0.15 m

Solution 1

(a)

Mark scheme

- angle (subtended at centre of a circle) when arc (length) $=$ radius **B1**

Solution 1

(b)(i)

Worked steps

- $v = r\omega$, so $\omega = \dfrac{v}{r} = \dfrac{17}{0.46}$
- $\omega = 37$ rad s^{-1}

Mark scheme

- v $=$ rω **C1**
- ω $=$ 17 / 0.46
- $=$ 37 rad s–1 **A1**

Solution 1

(b)(ii) Worked steps

The cog is fixed to the wheel, so both turn together: same period, same angular speed.

- $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{37}$
- $T = 0.17$ s

Solution 1

Mark scheme

- $T = 2\pi r / v$ or $T = 2\pi / \omega$ **C1**
- $= 2\pi \times 0.46 / 17$ or $2\pi / 37$
- $= 0.17$ s **A1**

Solution 1

(b)(iii)

Worked steps

In one full rotation, a point on the chain travels once around the cog.

- distance $= 2\pi r = 2\pi \times 0.038$
- $= 0.24$ m, as required.

The cog's own radius is what counts here — the chain wraps around the cog, not the wheel.

Mark scheme

- distance $= 2\pi \times 0.038 = 0.24$ m **A1**

Solution 1

(b)(iv) Worked steps

The two cogs mesh, so the **arc length** at the teeth is the same for both — that is the physical link between them, and it is what makes this one line of arithmetic.

- Angle in radians is $\theta = \frac{s}{r}$, arc length over radius.
- The arc length from (b)(iii) is 0.24 m, and the large cog's radius is 0.15 m.
- $\theta = \dfrac{0.24}{0.15} = 1.6$ rad

Solution 1

Divide by the **large** cog's radius: the arc is shared, so the bigger wheel turns through the smaller angle. And the answer is already in radians — the definition $\theta = s/r$ produces radians directly, so there is nothing to convert.
A check: 1.6 rad is about 92°, a quarter turn, which is what you would expect when the large cog's radius is a few times the small one's.

Mark scheme

- angle = arc length / radius **C1**
- = 0.24 / 0.15
- = 1.6 rad **A1**

Solution 1

(c)

Mark scheme

- point X moves through a smaller distance in the same time or
- (linear) speed of movement of point X / chain decreases or
- (linear) speed of (circumference of) both cogs decreases **B1**
- angular speed (of pedals) decreases **B1**

Question 1

9702/42 M25 ▪ Q1 ▪ 9 marks

1 A steel ball is placed on the inside surface of a hollow circular cone. The ball moves in a horizontal circle at constant speed, as shown in Fig. 1.1.

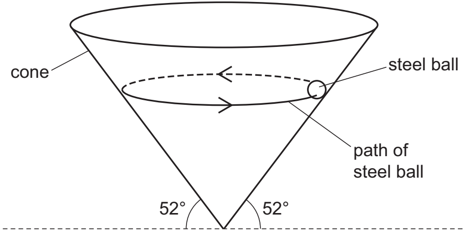


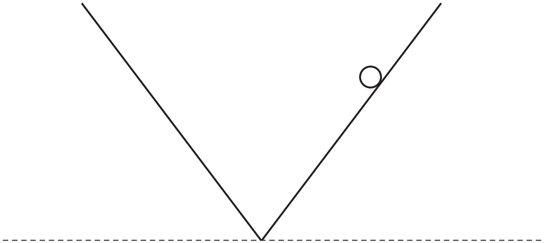
Fig. 1.1

The angle of the side of the cone to the horizontal is 52°. There is no friction between the ball and

Question 1

the cone.

(a) Fig. 1.2 shows a cross-section through the cone and the steel ball.



Question 1

Fig. 1.2

On Fig. 1.2, draw labelled arrows to show the **two** forces acting on the ball. [1]

(b) Describe how the forces acting on the ball cause its acceleration to be centripetal.

Question 1

(c) The ball moves in a circle of radius 0.15 m.

Show that the speed of the ball is 1.4 m s^{-1}.

Question 1

[3]

(d) Calculate the angular speed ω of the ball.

Question 1

(e) The speed of the ball is increased.

Explain why the radius of the circular path of the ball increases.

Question 1

[Total: 9]

Solution 1

(a)

Mark scheme

- arrow vertically downwards labelled ‘weight’ and arrow perpendicular to cone, inwards and upwards, labelled ‘normal
- contact force’

Solution 1

(b)

Mark scheme

- vertical component of contact force = weight (so no resultant force vertically)
- horizontal component of contact force is resultant force towards centre (of circle)

Solution 1

(c) Worked steps

Two forces act: the weight and the normal contact force from the cone. Their resultant is horizontal and provides the centripetal force.

- Resolving, the resultant acceleration is $a = g \tan 52°$.
- For circular motion, $a = \dfrac{v^2}{r}$.
- So $\dfrac{v^2}{0.15} = 9.81 \tan 52°$
- $v^2 = 0.15 \times 12.6 = 1.88$, giving $v = 1.4$ m s^{-1}, as required.

Solution 1

Mark scheme

- a = v2 / r
- a = g tan 52°
- 9.81 × tan 52° = v2 / 0.15 leading to v = 1.4 m s–1

Solution 1

(d)

Worked steps

- $v = r\omega$, so $\omega = \dfrac{v}{r} = \dfrac{1.4}{0.15}$
- $\omega = 9.3$ rad s^{-1}

Mark scheme

- v = rω or a = rω2
- ω = 1.4 / 0.15 or $\sqrt{}$(9.81 × tan 52° / 0.15)
- = 9.3 rad s–1

Solution 1

(e)

Worked steps

- The cone's angle is fixed, so the acceleration $g\tan 52^\circ$ is the **same** whatever the speed.
- With $a = \dfrac{v^2}{r}$ constant, v^2 is proportional to r.
- So a faster ball must move in a **larger** circle — it rides higher up the cone.

Mark scheme

- same resultant force / same acceleration so v2 is proportional to r (so if speed increases radius must also increase)