

Exercise walk-through

Scalars and vectors ■ 1.4

eXercise

You must be able to

- tell a **scalar** 标量 from a **vector** 矢量 and complete a classification table
- find the **resultant** 合矢量 of two **perpendicular** 垂直 vectors (size and direction)
- give the direction of $\vec{X} + \vec{Y}$ and $\vec{X} - \vec{Y}$
- find the resultant of **two equal forces** at an angle
- **resolve** 分解 any vector (a force *or* a velocity) into two perpendicular **components** 分量
- add three or more **coplanar** 共面 vectors by components
- **sketch** each component of a projectile's velocity against time

Method — A — Scalar or vector?

A scalar has size only; a vector has size **and** direction. **Quick test:** if “in which direction?” makes sense, it is a vector. In the exam you often tick a table:

Quantity	Scalar	Vector
speed	✓	
acceleration		✓
energy	✓	
momentum		✓

Method — A — Scalar or vector?

(Common scalars: mass, time, temperature, energy, distance, speed. Common vectors: displacement, velocity, acceleration, force, momentum.)

Method — B — Add two perpendicular vectors

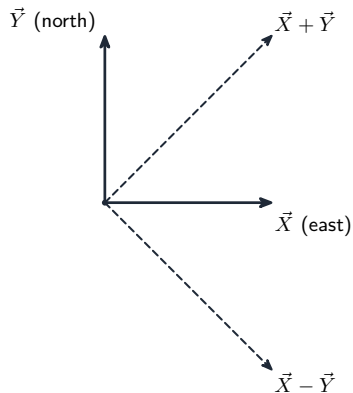
Draw them **tip to tail**; the resultant closes the triangle. For two vectors at 90° the size is $|\vec{R}| = \sqrt{X^2 + Y^2}$ and the direction is $\tan \theta = Y/X$.

Worked example. A swimmer heads north at 1.2 m s^{-1} ; the river flows east at 0.5 m s^{-1} :

$$|\vec{R}| = \sqrt{1.2^2 + 0.5^2} = 1.3 \text{ m s}^{-1}, \quad \tan \theta = \frac{0.5}{1.2} \Rightarrow \theta \approx 23^\circ \text{ east of north.}$$

Method — C — Direction of $\vec{X} + \vec{Y}$ and $\vec{X} - \vec{Y}$

To **add**, place the arrows tip to tail. To **subtract**, reverse the second arrow first: $\vec{X} - \vec{Y} = \vec{X} + (-\vec{Y})$, where $-\vec{Y}$ is the same size as $\vec{Y}$ but points the opposite way. With $\vec{X}$ east and $\vec{Y}$ north: $\vec{X} + \vec{Y}$ points **north-east**, and $\vec{X} - \vec{Y}$ points **south-east**.



Method — C2 — Adding three or more coplanar vectors

Coplanar 共面 vectors all lie in the same flat plane — every vector in this course is coplanar. For three or more, drawing triangles gets messy, so **use components**: add all the x -parts, add all the y -parts, then recombine.

Worked example. Three coplanar forces act at a point: 2.0 N east, 3.0 N north, 4.0 N west.

$$\sum x = 2.0 - 4.0 = -2.0 \text{ N}, \quad \sum y = +3.0 \text{ N}$$

$$R = \sqrt{2.0^2 + 3.0^2} = 3.6 \text{ N}, \quad \tan \theta = \frac{3.0}{2.0} \Rightarrow \theta = 56^\circ \text{ north of west}$$

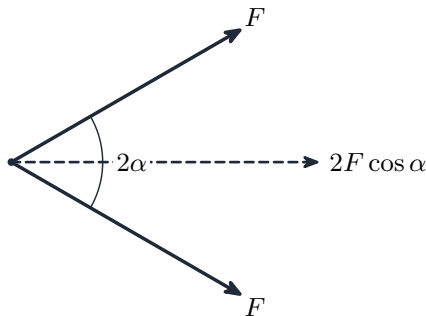
Take one direction as positive and stay with it — a lost minus sign is the usual error.

Method — D — Two equal forces at an angle

Two equal forces of size F with an angle 2α between them have a resultant of size $2F\cos\alpha$, along the line that **cuts the angle in half**.

Worked example. Two 6.0 N forces act with 60° between them. Here $2\alpha = 60^\circ$, so $\alpha = 30^\circ$:

$$R = 2F\cos\alpha = 2(6.0)\cos 30^\circ = 10.4 \text{ N.}$$



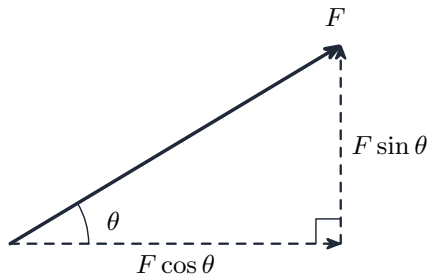
Method — E — Resolve a vector into components

Any vector $\vec{v}$ at angle θ to the horizontal splits into a horizontal part $v \cos \theta$ and a vertical part $v \sin \theta$ — this works for a force **or** a velocity 速度.

Force example. A 50 N force at 30° :
horizontal = $50 \cos 30^\circ = 43$ N, vertical
= $50 \sin 30^\circ = 25$ N.

Velocity example. A ball kicked at
 28 m s^{-1} , 34° above the horizontal:
 $v_H = 28 \cos 34^\circ = 23 \text{ m s}^{-1}$,
 $v_V = 28 \sin 34^\circ = 16 \text{ m s}^{-1}$.

On a slope of angle θ , the weight splits
into $W \sin \theta$ **along** the slope and $W \cos \theta$
into the slope.



Method — F — The two components of a projectile, against time

Once the components are found, each one has its own simple graph — and the exam asks you to sketch both on one grid:

- **horizontal** v_H : a **horizontal straight line** (no horizontal force, so it never changes).
- **vertical** v_V : a **straight line sloping down**, starting at $+v\sin\theta$, crossing zero at the top of the flight, and reaching $-v\sin\theta$ on landing (taking up as positive).

The gradient of that second line is $-g$, and the time to the top is v_V/g . Label each line, and use the whole time axis you are given.

Practice 2.1 ■ 3 marks

Complete the table by ticking **scalar** or **vector** for each quantity.

Quantity	Scalar	Vector
distance		
force		
temperature		
velocity		
power		
acceleration		

Solution 2.1

Method: A — Scalar or vector?

Worked steps

distance scalar; force vector; temperature scalar; velocity vector; power scalar; acceleration vector **B1** **B1** **B1** *for any four correct, for all six, no extra ticks.*

Practice 2.2 ■ 2 marks

Forces of 3.0 N east and 4.0 N north act at a point. Find the **magnitude** 大小 of the resultant.

Solution 2.2

Method: C2 — Adding three or more coplanar vectors

Worked steps

$$|\vec{R}| = \sqrt{3.0^2 + 4.0^2} = \sqrt{25} = 5.0 \text{ N} \quad \text{M1} \quad \text{A1}.$$

Practice 2.3 ■ 2 marks

Resolve a 50 N force acting at 30° to the horizontal into its horizontal and vertical components.

Solution 2.3

Method: E — Resolve a vector into components

Worked steps

Horizontal = $50 \cos 30^\circ = 43 \text{ N}$; vertical = $50 \sin 30^\circ = 25 \text{ N}$ **M1** **A1**.

Practice 2.4 ■ 2 marks

A ball leaves the ground at 20 m s^{-1} at 60° above the horizontal. Find the **horizontal** and **vertical** components of its velocity.

Solution 2.4

Method: E — Resolve a vector into components

Worked steps

$$v_H = 20 \cos 60^\circ = 10 \text{ m s}^{-1}; v_V = 20 \sin 60^\circ = 17 \text{ m s}^{-1} \quad \text{M1 A1}.$$

Practice 2.5 ■ 2 marks

Two forces, each 5.0 N, act at a point with 90° between them. Find the **magnitude** of the resultant.

Solution 2.5

Method: D — Two equal forces at an angle

Worked steps

$$R = \sqrt{5.0^2 + 5.0^2} = 7.1 \text{ N (or } 2F \cos 45^\circ = 2(5.0)(0.707) = 7.1 \text{ N)} \quad \text{M1 A1}.$$

Practice 2.6 ■ 3 marks

Three **coplanar** forces act at a point: 6.0 N east, 2.0 N west and 5.0 N north. Find the **magnitude** and the **direction** of the resultant.

Solution 2.6

Method: C2 — Adding three or more coplanar vectors

Worked steps

$\sum x = 6.0 - 2.0 = 4.0$ N east; $\sum y = 5.0$ N north (M1); $R = \sqrt{4.0^2 + 5.0^2} = 6.4$ N (A1); $\tan \theta = \frac{5.0}{4.0}$, so $\theta = 51^\circ$ north of east (A1).

Exam-style 3.1 ■ 1 mark

Which of these is a **vector** quantity?

A energy

B power

C displacement

D temperature

Solution 3.1

Method: A — Scalar or vector?

A energy

B power

C displacement



D temperature

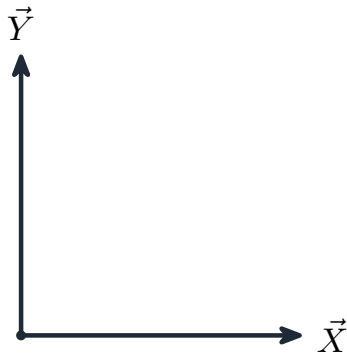
Worked steps

C — displacement.

Exam-style 3.2 ■ 1 mark

Two vectors $\vec{X}$ and $\vec{Y}$ are shown: $\vec{X}$ points east and $\vec{Y}$ points north. What are the directions of $\vec{X} + \vec{Y}$ and $\vec{X} - \vec{Y}$?

- A $\vec{X} + \vec{Y}$ north-east; $\vec{X} - \vec{Y}$ south-east
- B $\vec{X} + \vec{Y}$ north-east; $\vec{X} - \vec{Y}$ north-west
- C $\vec{X} + \vec{Y}$ south-west; $\vec{X} - \vec{Y}$ south-east
- D $\vec{X} + \vec{Y}$ south-east; $\vec{X} - \vec{Y}$ north-east



Solution 3.2

Method: C — Direction of $\vec{X} + \vec{Y}$ and $\vec{X} - \vec{Y}$

A $\vec{X} + \vec{Y}$ north-east; $\vec{X} - \vec{Y}$ south-east



B $\vec{X} + \vec{Y}$ north-east; $\vec{X} - \vec{Y}$ north-west

C $\vec{X} + \vec{Y}$ south-west; $\vec{X} - \vec{Y}$ south-east

D $\vec{X} + \vec{Y}$ south-east; $\vec{X} - \vec{Y}$ north-east

Worked steps

A — $\vec{X} + \vec{Y}$ points north-east; reversing $\vec{Y}$ (now south) gives $\vec{X} - \vec{Y}$ pointing south-east.

Exam-style 3.3 ■ 1 mark

Two forces of 6.0 N and 8.0 N act at a point at right angles to each other. What is the magnitude of the resultant force?

A 2.0 N

B 10 N

C 14 N

D 48 N

Solution 3.3

Method: D — Two equal forces at an angle

A 2.0 N

B 10 N



C 14 N

D 48 N

Worked steps

$$\text{B} — \sqrt{6.0^2 + 8.0^2} = \sqrt{100} = 10 \text{ N.}$$

Exam-style 3.4 ■ 2 marks

A stone is thrown with a velocity of 15 m s^{-1} at 40° above the horizontal.

(a) Find the **horizontal** and **vertical** components of this velocity.

(b) Air resistance is negligible. State which component stays **constant** during the flight, and why.

Solution 3.4

Method: E — Resolve a vector into components

Worked steps

(a) Horizontal = $15 \cos 40^\circ = 11 \text{ m s}^{-1}$; vertical = $15 \sin 40^\circ = 9.6 \text{ m s}^{-1}$ **M1** **A1**.

(b) The **horizontal** component stays constant **B1**, because with no air resistance there is no horizontal force, so no horizontal acceleration **B1**.

Exam-style 3.5 ■ 2 marks

Two forces, each 7.0 N, act at a point with 80° between them. Find the **magnitude** of the resultant.

Solution 3.5

Method: D — Two equal forces at an angle

Worked steps

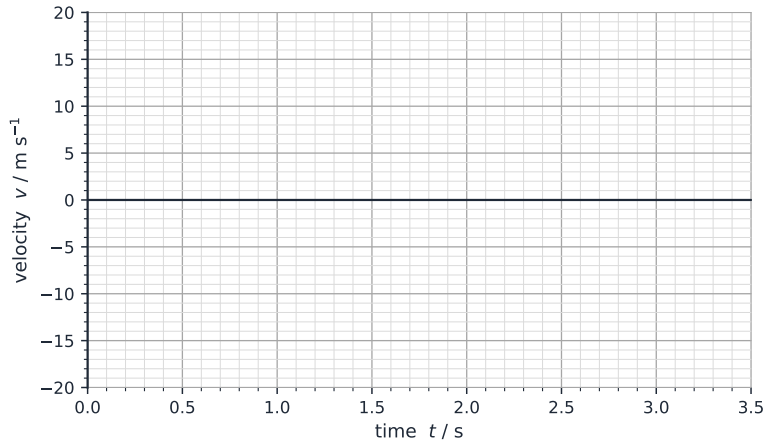
$$2\alpha = 80^\circ, \text{ so } \alpha = 40^\circ: R = 2F \cos \alpha = 2(7.0) \cos 40^\circ = 10.7 \text{ N} \quad \text{M1} \quad \text{A1}.$$

Exam-style 3.6 ■ 2 marks

A player kicks a ball from level ground with a velocity of 21 m s^{-1} at 55° to the horizontal. Air resistance is negligible.

- (a) **Calculate** the horizontal component v_H and the vertical component v_V of the velocity of the ball as it leaves the ground.
- (b) **Show that** the ball reaches its maximum height at time $t = 1.75 \text{ s}$.
- (c) The ball lands at time $t = 3.5 \text{ s}$. On the grid below, **sketch** the variation with time t of v_H and of v_V , from $t = 0$ to $t = 3.5 \text{ s}$. Take the upward direction as positive.
Label your lines H and V.
- (d) **Determine** the magnitude and the direction of the velocity of the ball just before it lands.
- (e) State why the horizontal component of the velocity does not change during the flight.

Exam-style 3.6 ■ 2 marks



Solution 3.6

Method: F — The two components of a projectile, against time

Worked steps

- (a) $v_H = 21 \cos 55^\circ = 12 \text{ m s}^{-1}$ **A1**; $v_V = 21 \sin 55^\circ = 17 \text{ m s}^{-1}$ **A1**.
- (b) At the top the vertical component is zero, so $t = \frac{v_V}{g} = \frac{17.2}{9.81} = 1.75 \text{ s}$ **A1**.
- (c) H: a horizontal straight line at $v = 12 \text{ m s}^{-1}$ across the whole grid, labelled H **B1**. V: a straight line of negative gradient, labelled V **B1**, starting at $+17 \text{ m s}^{-1}$ and ending at -17 m s^{-1} **B1**, crossing $v = 0$ at $t = 1.75 \text{ s}$ **B1**.
- (d) Just before landing $v_V = -17 \text{ m s}^{-1}$ and $v_H = 12 \text{ m s}^{-1}$ **M1**; magnitude $= \sqrt{12^2 + 17^2} = 21 \text{ m s}^{-1}$ **A1**; direction $\tan \theta = \frac{17}{12}$, so $\theta = 55^\circ$ **below** the horizontal

Solution 3.6

A1. (The speed and the angle match the launch — the flight is symmetric.)

(e) There is no horizontal force on the ball (air resistance is negligible), so there is no horizontal acceleration **B1**.