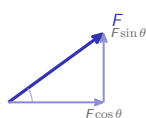


Mechanics

A-Level Mathematics • Topic 4

A-Level Mathematics



Mechanics

What we will cover

- 1 Forces & equilibrium
- 2 Kinematics
- 3 Momentum
- 4 Newton's laws
- 5 Energy work & power
- 6 Recap



1 Forces

Mechanics

└ Forces

Forces and equilibrium

Resolve every force into perpendicular components. At equilibrium the vector sum is zero:

$$\sum F_x = 0, \quad \sum F_y = 0.$$

A particle in equilibrium can be drawn as a closed vector triangle.

Mechanics

└ Forces

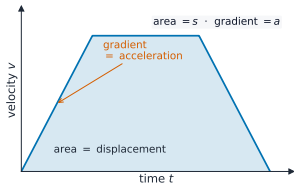
Friction

$F \leq \mu R$. Equality is limiting friction (on the point of slipping). μ is the coefficient of friction; R is the normal reaction.

Friction always opposes the (potential) slip. If the object is at rest and not about to move, F can be less than μR .

2 Kinematics and momentum

Kinematics



v - t graph: area = displacement,
gradient = acceleration.

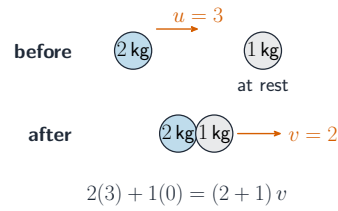
$$v = u + at, \quad s = ut + \frac{1}{2}at^2, \quad v^2 = u^2 + 2as.$$

Rest, $a = 2.5$, $t = 4$ s

$v = 10$ m/s, $s = 20$ m

The shaded area gives the distance travelled; the slope of the line gives the acceleration.

Momentum



Conservation of
momentum 动量守恒:

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2.$$

2 kg at 3 hits still 1 kg, stick

$$v = \frac{6}{3} = 2 \text{ m/s}$$

Worked example – a perfectly inelastic collision

A 2 kg body at 3 m s^{-1} hits a stationary 1 kg body and they stick together. Find the common speed.

■ conservation of linear
momentum 动量守恒:
 $m_1 u_1 = (m_1 + m_2)v$

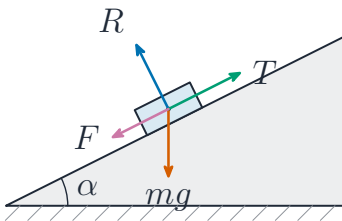
■ $2(3) = 3v$

■ $v = 2 \text{ m s}^{-1}$

Momentum is conserved;
kinetic energy is not.
Here 9 J before becomes
6 J after – the missing
3 J went into deformation
and heat.

3 Newton and energy

Newton's laws



$$F = ma, \quad W = mg.$$

On an inclined plane 斜面:
resolve along and perpendicular to
the slope, then apply $F = ma$.

Pull 12 kg up a 20° rough
slope

$$T \approx 110 \text{ N}$$

The laws, and the forces they name

Newton's laws of motion

Newton's laws of motion 牛顿运动定律: a body keeps its velocity unless a resultant 合力 force acts; $F = ma$; and every force has an equal and opposite partner.

Newton's third law

Newton's third law 牛顿第三定律 pairs act on different bodies – which is why they never cancel in one free-body diagram.

Connected particles

For connected particles 连接的物体 write $F = ma$ for each body separately, with the same acceleration and the tension appearing twice, in opposite directions.

Thrust

A light rod can push as well as pull: the compression force is thrust 推力, and a string can never provide it.

Limiting equilibrium 临界平衡 is the instant before sliding: friction is at its maximum μR , and the body is still in equilibrium.

Energy, work and power

$$W = Fd \cos \theta, \quad KE = \frac{1}{2}mv^2, \quad PE = mgh.$$

No friction $\Rightarrow$ total energy conserved. Power $P = Fv$.

12 kW engine at 20 m/s

$$F = \frac{P}{v} = 600 \text{ N}$$

The energy quantities you can calculate

kinetic energy	$\frac{1}{2}mv^2$
gravitational potential energy 重力势能	mgh , measured from a stated level
work done	Fd – force times distance along the force
power	$\frac{\text{work}}{\text{time}} = Fv$ for a constant force
conservation of energy 能量守恒	with no resistance, KE + PE is constant
with resistance	work done against air resistance 空气阻力 or friction is the energy lost from that total

On a **velocity-time graph** 速度时间图 the **area** is the displacement and the **gradient** is the acceleration, with a negative gradient meaning **deceleration** 减速.

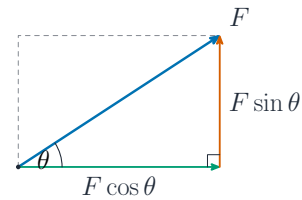
Forces and equilibrium, continued



Every mechanics problem is a picture like this. Three forces act on the climber – weight straight down, tension along the rope, and a push from the rock – and because they balance, the climber hangs still in equilibrium. Resolving each into horizontal and vertical parts turns the picture into equations

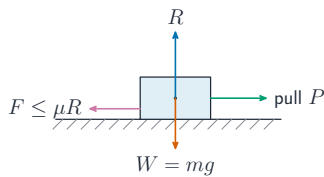
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Forces and equilibrium, continued (2)



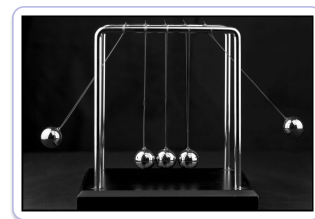
A force at angle θ has a horizontal part $F \cos \theta$ and a vertical part $F \sin \theta$.

Friction, continued



On a rough surface the contact force splits into the normal reaction R and friction F (at most μR).

Momentum, continued



A Newton's cradle demonstrates conservation of momentum in collisions.

Exam tips

- Draw a clear **force diagram** and resolve into perpendicular components; for equilibrium, each direction sums to zero.
- Use **SUVAT** only for constant acceleration and keep a consistent positive direction.
- Apply $F = ma$ along the direction of motion, including friction ($F = \mu R$) on a rough surface.
- State your **assumptions** (light inextensible string, smooth pulley, particle) – they are often worth a mark.

4

Recap

Topic 4 – key takeaways

1 Forces

resolve into components; $F \leq \mu R$

2 Kinematics

suvat; v - t area & gradient

3 Newton

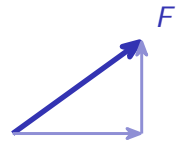
$F = ma$; resolve on a slope

4 Energy

$\frac{1}{2}mv^2$, mgh ; $P = Fv$

Check yourself

- 1 Maximum friction before slipping?
- 2 What does the area under a v - t graph give?
- 3 Driving force of a 10 kW engine at 25 m/s?
- 4 Quantity conserved in a collision?



Answers 1. μR 2. displacement 3. $P/v = 400 \text{ N}$ 4. momentum