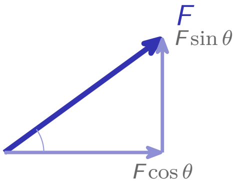
Mechanics

A-Level Mathematics ▪ Topic 4

A-Level Mathematics



What we will cover

1. Forces & equilibrium
2. Kinematics
3. Momentum
4. Newton's laws
5. Energy work & power
6. Recap

1 Forces

Forces and equilibrium

Resolve every force into perpendicular components. At equilibrium the vector sum is zero:

$$\sum F_x = 0, \qquad \sum F_y = 0.$$

> A particle in equilibrium can be drawn as a closed vector triangle.

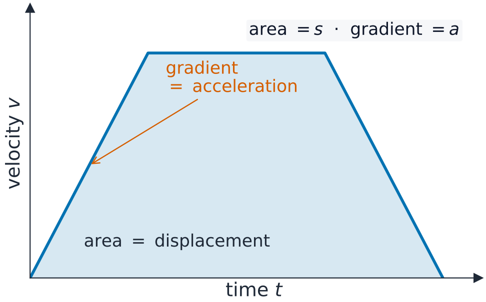
Friction

$F \leq \mu R$. Equality is limiting friction (on the point of slipping). μ is the coefficient of friction; R is the normal reaction.

> Friction always opposes the (potential) slip. If the object is at rest and not about to move, F can be less than μR.

2 Kinematics and momentum

Kinematics



v–t graph: area $=$ displacement, gradient $=$ acceleration.

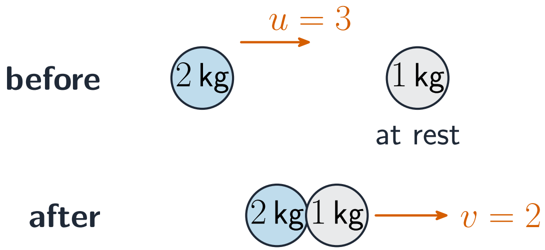
$$v = u + at, \quad s = ut + \tfrac{1}{2}at^2, \quad v^2 = u^2 + 2as.$$

Rest, $a = 2.5$, $t = 4\,\mathrm{s}$

$v = 10\,\mathrm{m/s}$, $s = 20\,\mathrm{m}$

> The shaded area gives the distance travelled; the slope of the line gives the acceleration.

Momentum



$$2(3) + 1(0) = (2 + 1)\, v$$

Conservation of momentum 动量守恒:

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2.$$

$2\,\mathrm{kg}$ at 3 hits still $1\,\mathrm{kg}$, stick

$$v = \frac{6}{3} = 2\,\mathrm{m/s}$$

Worked example – a perfectly inelastic collision

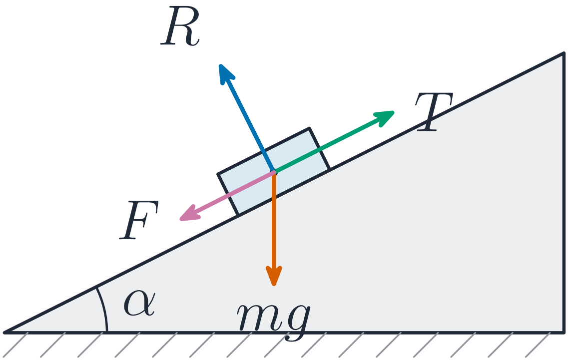
A 2 kg body at 3 m s^{-1} hits a stationary 1 kg body and they stick together. Find the common speed.

- **conservation of linear momentum** 动量守恒: $m_1 u_1 = (m_1 + m_2) v$
- $2(3) = 3v$
- $v = 2$ m s^{-1}

Momentum is conserved; kinetic energy is not. Here 9 J before becomes 6 J after – the missing 3 J went into deformation and heat.

3 Newton and energy

Newton's laws



$$F = ma, \qquad W = mg.$$

On an **inclined plane** 斜面: resolve along and perpendicular to the slope, then apply $F = ma$.

Pull $12\,\mathrm{kg}$ up a 20° rough slope

$T \approx 110\,\mathrm{N}$

The laws, and the forces they name

Newton's laws of motion	**Newton's laws of motion** 牛顿运动定律: a body keeps its velocity unless a **resultant** 合力 force acts; $F = ma$; and every force has an equal and opposite partner.
Newton's third law	**Newton's third law** 牛顿第三定律 pairs act on *different* bodies – which is why they never cancel in one free-body diagram.
Connected particles	For **connected particles** 连接的物体 write $F = ma$ for each body separately, with the same acceleration and the tension appearing twice, in opposite directions.
Thrust	A light rod can push as well as pull: the compression force is **thrust** 推力, and a string can never provide it.

Limiting equilibrium 临界平衡 is the instant before sliding: friction is at its maximum μR, and the body is still in equilibrium.

Energy, work and power

$$W = Fd\cos\theta, \quad \mathrm{KE} = \tfrac{1}{2}mv^2, \quad \mathrm{PE} = mgh.$$

No friction $\Rightarrow$ total energy conserved. Power $P = Fv$.

$12\,\mathrm{kW}$ engine at $20\,\mathrm{m/s}$

$$F = \frac{P}{v} = 600\,\mathrm{N}$$

The energy quantities you can calculate

kinetic energy	$\frac{1}{2}mv^2$
gravitational potential energy 重力势能	mgh, measured from a stated level
work done	Fd – force times distance along the force
power	$\frac{\text{work}}{\text{time}} = Fv$ for a constant force
conservation of energy 能量守恒	with no resistance, KE + PE is constant
with resistance	work done against **air resistance** 空气阻力 or friction is the energy lost from that total

On a **velocity-time graph** 速度时间图 the area is the displacement and the gradient is the acceleration, with a negative gradient meaning **deceleration** 减速.

Forces and equilibrium, continued

Every mechanics problem is a picture like this. Three forces act on the climber – weight straight down, tension along the rope, and a push from the rock – and because they balance, the climber hangs still in equilibrium. Resolving each into horizontal and vertical parts turns the picture into equations

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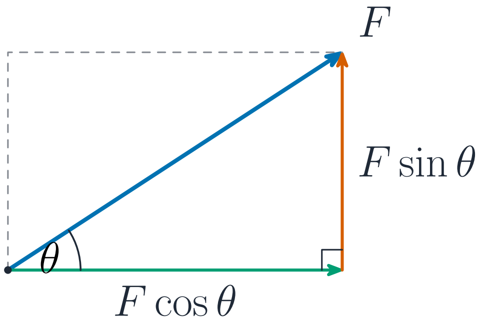
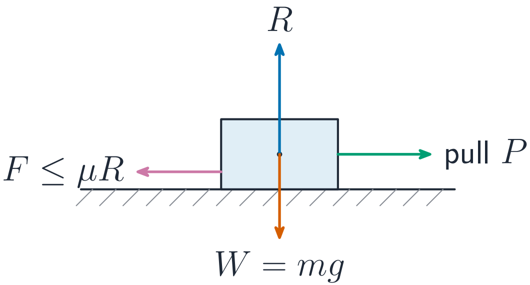
Forces and equilibrium, continued (2)



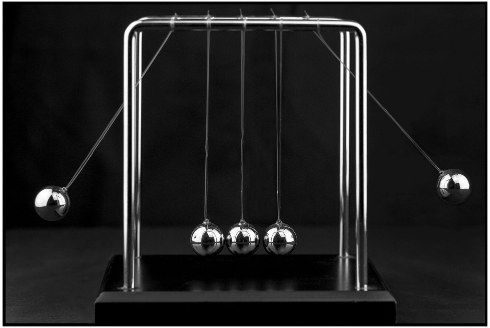
> A force at angle θ has a horizontal part $F\cos\theta$ and a vertical part $F\sin\theta$.

Friction, continued



On a rough surface the contact force splits into the normal reaction R and friction F (at most μR).

Momentum, continued



A Newton's cradle demonstrates conservation of momentum in collisions.

Exam tips

- Draw a clear **force diagram** and resolve into perpendicular components; for equilibrium, each direction sums to zero.
- Use **SUVAT** only for constant acceleration and keep a consistent positive direction.
- Apply $F = ma$ along the direction of motion, including friction ($F = \mu R$) on a rough surface.
- State your **assumptions** (light inextensible string, smooth pulley, particle) – they are often worth a mark.

4 Recap

Topic 4 – key takeaways

1 Forces

resolve into components; $F \leqslant \mu R$

2 Kinematics

suvat; v–t area & gradient

3 Newton

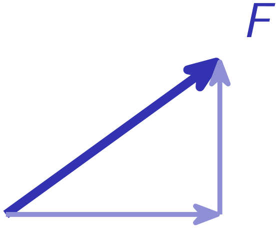
$F = ma$; resolve on a slope

4 Energy

$\frac{1}{2}mv^2$, mgh; $P = Fv$

Check yourself

1. Maximum friction before slipping?
2. What does the area under a v–t graph give?
3. Driving force of a $10\,\mathrm{kW}$ engine at $25\,\mathrm{m/s}$?
4. Quantity conserved in a collision?



Answers **1.** μR **2.** displacement **3.** $P/v = 400\,\mathrm{N}$ **4.** momentum