6 The Poisson distribution – Part 1

Name: _____ Class: _____ Date: _____

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	**中文**	**Write it 3 times** (English)		
Poisson distribution	泊松分布			
mean	均值			
variance	方差			

Recall

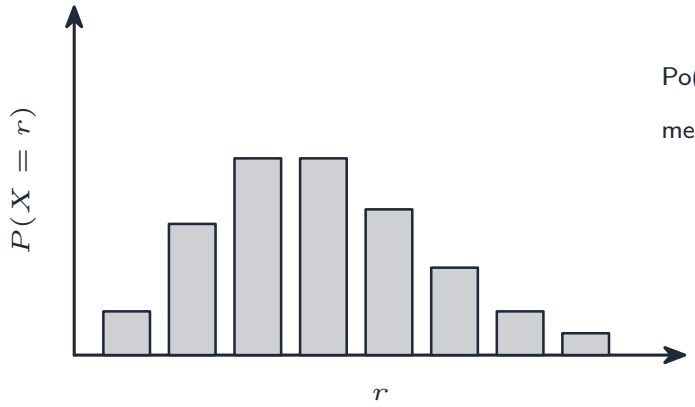
Statistics 2 adds the Poisson model for random events, and rules for combining random variables.

New ideas

Read these first, then do the Practice.

■ 1 The Poisson distribution

The **Poisson distribution** 泊松分布 $X \sim \text{Po}(\lambda)$ models the number of random events in a fixed interval, at an average rate λ:



Po(3):

mean = variance = 3

$$P(X = r) = e^{-\lambda}\frac{\lambda^r}{r!}$$

For a Poisson variable the **mean** 均值 and the **variance** 方差 both equal λ.

2 Combining random variables

$$E(aX + b) = aE(X) + b$$

$$\mathrm{Var}(aX + b) = a^2\,\mathrm{Var}(X)$$

A constant added shifts the mean but not the variance; a multiplier scales the mean by a and the variance by a^2.

Watch out: for a Poisson variable, *both* the mean and the variance equal λ. When transforming: $E(aX+b) = aE(X)+b$, but $\mathrm{Var}(aX+b) = a^2\,\mathrm{Var}(X)$ — the constant b drops out of the variance, and the multiplier is *squared*.

Practice

Try these in order.

6.1 What does the Poisson distribution model? [1]

6.2 For $X \sim \text{Po}(\lambda)$, what is the variance? [1]

6.3 $X \sim \text{Po}(3)$. Use the formula to find $P(X = 2)$. (Take $e^{-3} = 0.0498$.) [2]

6.4 X has mean 5. Find $E(3X - 1)$. [1]

6.5 X has variance 4. Find $\text{Var}(3X - 1)$. [2]

6.6 Challenge. Calls arrive at a help-desk at an average of 4 per hour, modelled by Po(4). Find the probability of exactly 5 calls in an hour. (Take $e^{-4} = 0.0183$.) [3]