5 Probability & Statistics 1 – Part 2

Name: _____ Class: _____ Date: _____

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	**中文**	**Write it 3 times** (English)		
probability	概率			
independent events	独立事件			
mutually exclusive events	互斥事件			
discrete random variable	离散型随机变量			
expectation	期望			
binomial distribution	二项分布			

Recall

In **Part 1** you met averages, spread, and how to count arrangements (permutations and combinations). Part 2 looks at **combining probabilities**, and at **random variables**.

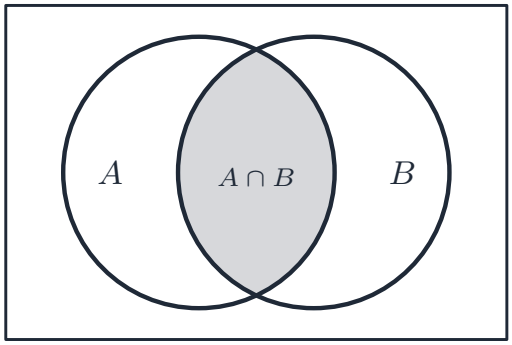
New ideas

■ 1 Probability rules

Find a **probability** 概率 by counting equally likely outcomes. Combine probabilities with these rules:

- **addition** (for "or"): $P(A \cup B) = P(A) + P(B) - P(A \cap B)$;
- **multiplication** (for "and", when events are **independent events** 独立事件): $P(A \cap B) = P(A)\,P(B)$.

Two events are **mutually exclusive events** 互斥事件 if they cannot both happen (then $P(A \cap B) = 0$).



Worked example. $P(A) = 0.5$, $P(B) = 0.4$, $P(A \cap B) = 0.2$. Since $0.5 \times 0.4 = 0.2 = P(A \cap B)$, the events are **independent**.

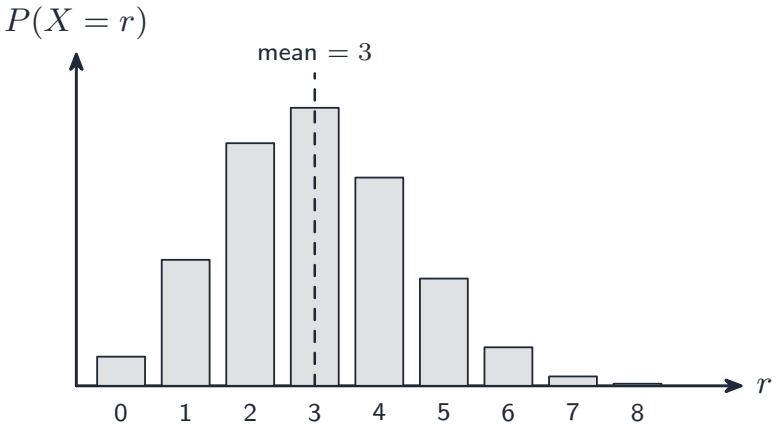
Micro-check. A fair die is rolled. Find the probability of getting a 5. [1]

■ 2 Discrete random variables

A **discrete random variable** 离散型随机变量 X takes separate values, each with a probability (the probabilities add to 1). Its **expectation** 期望 (mean) is

$$E(X) = \sum x\, P(X = x).$$

The **binomial distribution** 二项分布 $X \sim B(n, p)$ counts the successes in n independent trials, with $E(X) = np$.



Worked example. For $X \sim B(10, 0.3)$: $E(X) = np = 10 \times 0.3 = 3$.

Micro-check. For $X \sim B(20, 0.5)$, find $E(X)$. [1]

> **Watch out:** for "or" use $P(A \cup B) = P(A) + P(B) - P(A \cap B)$; for independent "and" use $P(A \cap B) = P(A)P(B)$. Test independence by checking whether $P(A)P(B) = P(A \cap B)$. For $X \sim B(n, p)$, $E(X) = np$.

Practice

5.1 A fair die is rolled. Find the probability of getting an **even** number. [1]

5.2 $P(A) = 0.3$, $P(B) = 0.5$, $P(A \cap B) = 0.1$. Find $P(A \cup B)$. [2]

5.3 $P(A) = 0.4$, $P(B) = 0.25$, and $P(A \cap B) = 0.1$. Test whether A and B are **independent**. [2]

5.4 A random variable X has $P(X = 1) = 0.2$, $P(X = 2) = 0.5$, $P(X = 3) = 0.3$. Find $E(X)$. [2]

5.5 For $X \sim B(40, 0.25)$, find $E(X)$. [1]

5.6 Challenge. A fair coin is tossed 6 times. Using the binomial distribution, find the

expected number of heads and the probability of getting exactly 6 heads. [3]