

3 Partial fractions and binomials – Part 1

Name: _____ Class: _____ Date: _____

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)
partial fractions	部分分式	_____
binomial expansion	二项展开式	_____

Recall

Pure 3 adds tools for harder algebra. Two of them are partial fractions and the binomial expansion for any power.

New ideas

Read these first, then do the Practice.

■ 1 Partial fractions

A single fraction with a factorised bottom can be split into simpler fractions — **partial fractions** 部分分式:

$$\frac{x+4}{(x+1)(x-2)} \xrightarrow{\text{split}} \frac{2}{x-2} - \frac{1}{x+1}$$

Write $\frac{x+4}{(x+1)(x-2)} = \frac{A}{x+1} + \frac{B}{x-2}$, then find A and B .

Worked example. Multiplying out: $x+4 = A(x-2) + B(x+1)$. Put $x=2$: $6 = 3B$, so $B=2$. Put $x=-1$: $3 = -3A$, so $A=-1$. Hence it equals $\frac{-1}{x+1} + \frac{2}{x-2}$.

■ 2 The binomial expansion for any power

The **binomial expansion** 二项展开式 also works for a fraction or a negative power, when $|x| < 1$:

$$(1+x)^{1/2} \\ = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \dots \quad (\text{only if } |x| < 1)$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots$$

Watch out: for a fractional or negative n , this expansion is only valid when $|x| < 1$. For $(a+x)^n$, first take out a : $a^n \left(1 + \frac{x}{a}\right)^n$. In partial fractions, substitute clever values of x to find A and B quickly.

Practice

Try these in order.

3.1 What does it mean to write a fraction in *partial fractions*? [1]

3.2 In $\frac{x+4}{(x+1)(x-2)} = \frac{A}{x+1} + \frac{B}{x-2}$, so $x+4 = A(x-2) + B(x+1)$. Put $x=2$ to find B . [2]

3.3 The binomial expansion of $(1+x)^n$ for a fractional power is valid only when...? [1]

3.4 Write down the first three terms of $(1+x)^n$. [1]

3.5 Using $n = \frac{1}{2}$, what is the second term of $(1+x)^{1/2}$? [1]

3.6 Challenge. Find the first three terms of the expansion of $(1+2x)^{1/2}$, and state for

which x it is valid.

[3]