1 Pure Mathematics 1 – Part 1

Name: ____________ Class: ________ Date: __________

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	**中文**	**Write it 3 times** (English)		
factorising	因式分解			
parabola	抛物线			
quadratic	二次式			
coefficients	系数			
Completing the square	配方			
vertex	顶点			
discriminant	判别式			
real roots	实根			

Recall

At IGCSE you worked with quadratics. Bring this with you:

- You solved quadratic equations by **factorising** 因式分解.
- You drew the **parabola** 抛物线 — the U-shaped curve of $y = ax^2 + bx + c$.

At AS Pure 1 we add a tidy form that reveals the curve's turning point, and a quick test for how many solutions an equation has.

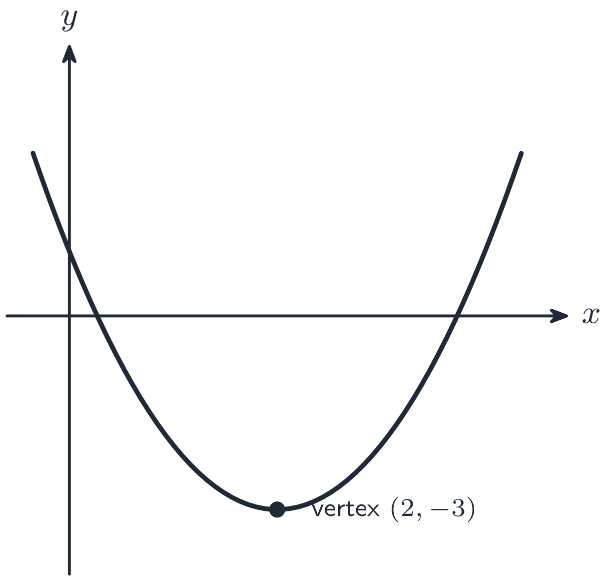
New ideas

1 Completing the square

A **quadratic** 二次式 has the form ax^2+bx+c; the numbers a, b, c are its **coefficients** 系数.

Completing the square 配方 rewrites it as $a(x+p)^2+q$. This form shows the **vertex** 顶点 (turning point) of the curve at $(-p,\ q)$.

Worked example. $x^2-4x+1=(x-2)^2-4+1=(x-2)^2-3$. The vertex is at $(2,-3)$.



Micro-check. Write x^2-6x in the form $(x+p)^2+q$. [1]

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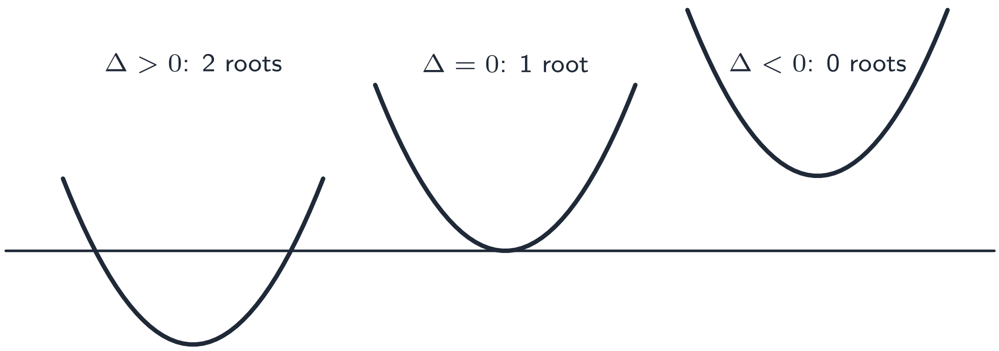
2 The discriminant

The **discriminant** 判别式 of ax^2+bx+c is $\Delta=b^2-4ac$. It tells you how many **real roots** 实根 (real solutions) the equation $ax^2+bx+c=0$ has:

- $b^2-4ac>0$ — two real roots;
- $b^2-4ac=0$ — one (repeated) real root;
- $b^2-4ac<0$ — no real roots.



Worked example. For $x^2 - 4x + 1$: $\Delta = (-4)^2 - 4(1)(1) = 16 - 4 = 12 > 0$, so there are **two** real roots.

Micro-check. Find the discriminant of $x^2 + 2x + 5$, and state the number of real roots. [2]

Watch out: to complete the square, halve b and square it. In $a(x+p)^2+q$ the vertex is at $(-p,\ q)$ — note the *sign flip* on p. For the discriminant: $b^2 - 4ac > 0 \to$ two roots, $= 0 \to$ one, $< 0 \to$ none.

Practice

1.1 Write $x^2 - 8x + 3$ in the form $(x + p)^2 + q$. [2]

1.2 State the vertex of the curve $y = (x - 3)^2 - 4$. [1]

1.3 Find the discriminant of $2x^2 + 3x + 1$, and state how many real roots the equation $2x^2 + 3x + 1 = 0$ has. [2]

1.4 State the number of real roots of $x^2 - 4x + 4 = 0$ (use the discriminant). [2]

1.5 Solve $x^2 - 5x + 6 = 0$ by factorising. [2]

1.6 Challenge. Find the values of k for which $x^2+kx+9 = 0$ has exactly one (repeated) real root. [3]